International Trade and the Allocation of Capital Within Firms

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Abstract

This paper introduces an internal capital market into a two-factor model of multi-segment firms that features managers' empire building and informational frictions within the organization. Our novel theory shows that international trade imposes discipline on divisional managers and improves capital allocation across divisions, thereby lowering the conglomerate discount. The theory can explain why exporters exhibit a lower conglomerate discount than non-exporters (a new fact we establish). We exploit the China shock as an exogenous change to competition to confirm the model's predictions with data on US companies.

JEL classification: F12, G30, L22, D23.

Keywords: multi-product firms, trade and organization, internal capital markets, conglomerate discount, China shock.

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1 Introduction

A well-established observation in the trade literature is that conglomerate firms are more productive than single-product firms and dominate in international trade and manufacturing sales. They account for two-thirds of exporters, 98% of export value, and 91% of US manufacturing sales (Schoar, 2002; Bernard et al., 2018). These facts appear to be at odds with findings on the 'conglomerate discount' indicating that multi-segment firms have lower a Tobin's Q than single-product firms and trade at a discount (Lang and Stulz, 1994; Ozbas and Scharfstein, 2010)

This paper reconciles these seemingly conflicting views by embedding an internal capital market into a model of multi-product firms with monopolistic competition. In the model, managers of multi-product firms compete for funds within their firms. The headquarters does not know the true marginal costs of its divisions, while the divisional managers do. This informational friction allows managers to run bigger divisions by over-reporting actual marginal costs to receive more capital. Managers in better divisions have a greater scope for mis-reporting, as they are less at risk of not being financed at all. The result is a distorted allocation of capital across divisions: headquarters over-allocate capital to better divisions. This reduces firms' return on assets and depresses their Tobin's Q, resulting in a conglomerate discount.

We then introduce international trade into the model and investigate how it affects the allocation of capital within firms. Fiercer competition lowers the cost level at which firms and their divisions can survive in the market. Because managers use this cut-off cost level as a benchmark when deciding by how much to over-report costs, competition reduces their scope for over-reporting. The model thus predicts that competition has a disciplining effect. It leads to a re-allocation of capital within multi-segment firms, thereby increasing firms' profitability and lowering the conglomerate discount. We confirm the model's predictions in the data by exploiting exogenous variation in import penetration from China as a shock to competition.

Traditional models of multi-segment firms in the trade literature predict that project funding goes to the most productive segments and that all projects with positive profits are financed.[1] However, an internal capital market within an organisation is subject to informational frictions – it may not always allocate resources efficiently and it may not

[1]See, for example, Bernard et al. (2010); Eckel and Neary (2010); Dhingra (2013); Mayer et al. (2014); Nocke and Yeaple (2014).

fund all projects with positive returns. To understand the type of products that firms finance, produce, and export – and how these decisions are shaped by competition – it is essential to examine the internal allocation of funds.[2] This requires a theory of multi-segment firms that is micro-founded in a finance theory of organization. Our paper provides a first step toward such a theory.

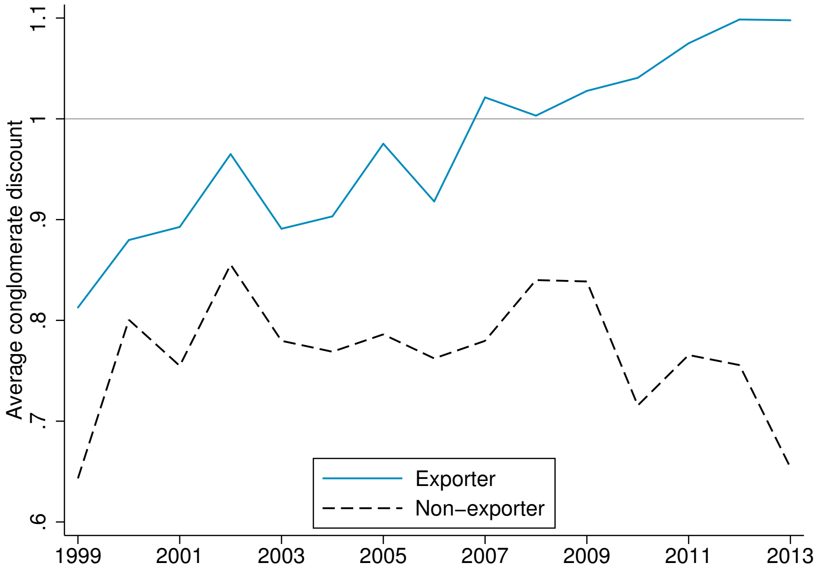
We incorporate an internal capital market into a two-factor version of the Mayer et al. (2014) monopolistic competition model of multi-product firms. In our model, divisional managers compete for funds allocated by the headquarters, subject to an informational asymmetry between the headquarters and divisional managers of firms. To allocate funds to the firm's various divisions, the headquarters ranks divisions relative to one another by their return and allocates more capital to the best-ranked segments. That is, it engages in "winner picking" (Stein, 1997). However, "over-investment" (Rajan et al., 2000; Scharfstein and Stein, 2000) to some divisions can arise because the headquarters knows less about a segment's true cost than its divisional managers. This information asymmetry allows firms' divisional managers, who have an appetite for running bigger divisions (i.e. they are empire-builders), to not report their costs truthfully. Instead, they over-report their costs and end up receiving more capital than optimal. As better divisions are less at risk of not being financed at all, they have more room to over-report.

In this set-up, we then investigate the effects of international trade on the efficiency of firms' internal capital markets. One novel empirical fact we establish is that exporters exhibit a lower conglomerate discount than non-exporting firms. Figure 1 plots the ratio of average Q of multi-segment firms over average Q of single-segment firms (the conglomerate discount). We split the sample into exporting and non-exporting firms. Our first observation is that multi-segment firms have lower Tobin's Q than single-segment firms. For both exporting and non-exporting firms, relative Q is below unity on average. The difference in Q represents the well-known conglomerate discount. The second observation is that the conglomerate discount is smaller for exporting firms (3%) than non-exporting firms (23%). Exporters' significantly lower conglomerate discount suggests that firms exposed to open markets differ from domestically active conglomerates. Furthermore, the discount remains stable for non-exporters but declines over time for exporters (higher values on the y-axis denote a narrowing in the difference in Q between single- and multi-

[2] Internal capital markets represent an important source of funding in US publicly listed firms. Internal cash flow accounts for 83% of total funds in multi-segment firms. While it is equally important for exporters (80%), cash flow matters less in single-segment firms (30%); see also Marin and Schnitzer (2011).

segment firms). For the years following the financial crisis of 2007–08, the ratio surpasses 1 and the discount turns into a premium.

Figure 1: **Conglomerate discount by exporter status**



Note: This Figure plots the ratio of the average Q of multi-segment firms over the average Q of single-segment firms (the conglomerate discount). We split the sample into exporting and non-exporting firms. The patterns are robust to the accounting for firm characteristics, as well as time-varying trends at the industry level (unreported). Data are from Worldscope.

The key insight that we gain from introducing an internal capital market into the theory of multi-product firms is to highlight how external competition, which we model as a shock to the costs of international trade, affects the allocation of capital within firms. Specifically, tougher competition lowers the cost level at which firms and their divisions can survive in the market. As managers use this cut-off cost level as a benchmark when deciding by how much to over-report costs to get funds from the headquarters, they run a risk of not being financed if reported costs are too high. Consequently, tougher competition leaves less room for over-reporting and thereby reduces the conglomerate discount. In particular, assets increase relatively more in the best segments, in which managers' scope for over-reporting of costs declines the most (i.e., in those segments with the largest initial over-reporting).

The second part of the paper tests the model's predictions with detailed data on public US manufacturing firms at the segment level from 1999–2007. We exploit the increase in import penetration from China – the China shock – as a source of exogenous variation in industry-level competition. Following (Autor et al., 2013), we instrument US imports from China with Chinese imports in eight other advanced economies.

First, we establish that multi-segments firms allocate more capital to segments than single-segment firms with comparable costs. This over-allocation of capital moreover is especially pronounced among firms in which informational frictions are more severe. Theses results are consistent with one of our main model's predictions: mis-allocation within multi-segment firms arises because of cost over-reporting by segment managers due to informational frictions with headquarters, while no such incentive conflict exist in single-segment firms (where manager and owner are the same person).

Second, we show that a rise in import competition significantly reduces the conglomerate discount by disciplining the internal capital market. In terms of magnitude, a one standard deviation increase in import penetration in a given industry lowers the average conglomerate discount by around one-third over the sample period. In line with the channel highlighted by the model, we find a significant increase in allocated assets to the best segments in response to higher competition, relative to worse segments. In terms of magnitude, a one standard deviation increase in import penetration leads to a relative increase in assets allocated to the best segments by around 18%.

Given that the underlying friction giving rise to mis-allocation is asymmetric information, we expect the disciplining effects of import competition on segment assets to be particularly strong within firms that suffer more from informational asymmetries. To this end, we use data on CEO backgrounds to classify firms into those with high and low informational frictions. We categorize firms by the time their average CEO spent on the board. The longer the time a CEO has served on the board of a company, the more likely she is to know its segments and the lower the scope for over-reporting by divisional managers. We find the disciplining effects of competition (i.e., an increase in allocated assets to the best segments) to be particularly strong in firms subject to higher informational frictions. We confirm these findings with alternative measures.

We perform additional tests to rule out alternative explanations. First, we directly control for the cost of external capital, as declining interest rates have been linked to capital mis-allocation (Gopinath et al., 2017). Second, we account for the effects of changes in industry-level financing costs, proxied by the average interest expense over total debt for each industry. Finally, we show that our results are robust to the inclusion of fixed effects at the industry*year level that absorb any unobservable time-varying shocks at the industry level.

Our paper contributes to the literature in international trade. Workhorse models in the spirit of Mayer et al. (2014), including Bernard et al. (2010, 2011), Eckel and Neary (2010), Dhingra (2013), and Mayer et al. (2021), mostly abstract from financial issues within the firm. By introducing an internal capital market into a model of multi-segment firms we are able to endogenize the cost structure of multi-segment firms, which is typically exogenous in trade models of multi-segment firms. In our set-up, the cost structure of a firm becomes a distinct feature of the firms' internal organisation. This allows us to reconcile the conflicting views in the trade and finance literature of why multi-segment firms may sometimes be less or more productive than single segment firms. We show that the efficiency of multi-segment firms depends on the competition for funds in the internal capital market, the severity of informational frictions within the organization, as well as on the degree of competition in the output market.

We also speak to the literature that introduces firms' internal organization into trade models. Marin and Verdier (2012, 2014) and Marin et al. (2021) introduce the authority-based hierarchy model of the firm of Aghion and Tirole (1997) and Caliendo and Rossi-Hansberg (2012), while Caliendo et al. (2015, 2020) and Friedrich (2022) bring knowledge-based hierarchies into international trade. These papers show that firms reorganize to a more decentralized organization in a more competitive trade environment. Related, Bloom et al. (2012) and Guadalupe and Wulf (2010) empirically find that more competitive markets lead firms to decentralize and to introduce flatter firm hierarchies, while Grossman and Helpman (2002) and Conconi et al. (2012) show that international trade may lead firms to move activity outside of their boundaries. None of these papers examine, however, how trade affects the allocation of capital within the firm, nor its implications for the cost structure of multi-segments firms.

The rest of this paper is organised as follows. Sections 2 to 4 present the model. Section 5 derives testable predictions and then takes the model to the data: exploiting the 'China shock' as an exogenous shock to competition, it investigates the causal effect of competition on the allocation of capital within publicly listed US firms, as well as on the conglomerate discount. Finally, Section 6 concludes.

2 Setup of the model

We develop a model of heterogeneous firms in which single-segment firms (SSF) and multi-segment firms (MSF) coexist in a market with monopolistic competition. A SSF has a simple organisation in which the firm's owner has full control over production of one type of good. A MSF is a conglomerate operating multiple unrelated segments. The owner has control over the core segment, but delegates control over non-core segments to divisional managers.

We build on the literature on multi-product firms with flexible technology as in Bernard et al. (2010), Eckel and Neary (2010), Dhingra (2013), and Nocke and Yeaple (2014)). Each firm has a core competence in producing one core good. The firm can introduce other non-core goods that have higher production costs than the core good, as they require different know-how than the firm's core competence. Our goal is to model the allocation of capital across divisions of a conglomerate firm operating several vertically unrelated segments. We abstract therefore from substitution patterns among products of the same firm, and we develop our framework along the lines of Mayer et al. (2014) that does not hinge on the so-called *cannibalisation effect*.

2.1 Endowments and preferences

A country is endowed with a fixed stock of capital K and populated by a continuum of households of measure L. Each household is endowed with one indivisible unit of time, which is supplied either as labor or managerial effort.

Households have identical preferences over a continuum of varieties of horizontally differentiated goods of measure V and one homogeneous outside good. Their utility function is given by:

$$U = q_o^c + \alpha \int_0^V q_v^c dv - \frac{\beta}{2} \left(\int_0^V q_v^c dv \right)^2 - \frac{\gamma}{2} \int_0^V \left(q_v^c \right)^2 dv, \tag{1}$$

where q_o^c represents the consumption of the homogeneous outside good, while q_v^c represents the consumption of variety v of a differentiated good.[3]

[3]For a detailed discussion on the properties of this utility function, see Melitz and Ottaviano (2008). Parameters $\alpha > 0$ and $\beta > 0$ account for the substitution patterns between homogeneous goods and varieties of differentiated goods. The willingness to smooth consumption across differentiated goods increases with the parameter $\gamma > 0$, where higher γ implies lower substitution.

Households maximise their utility given by Equation (1) subject to the budget constraint $p_o q_o^c + \int_0^V p_v q_v^c dv \leq I^c$, taking the prices of the outside good p_o and varieties p_v as given. I^c denotes household's income. We outline the derivation of the consumer's problem in the appendix.

As in Melitz and Ottaviano (2008) we assume that the outside good is always consumed, $q_o^c > 0$. Thus, the outside sector absorbs residual expenditure (and resources) not allocated to the differentiated good sector. Demand for a differentiated good is zero above a certain choke price, which is an endogenous outcome in general equilibrium.

2.2 Production

The outside sector is competitive. The production of outside goods combines labor and capital as perfect substitutes, with the marginal productivity of labor equal to 1 and the marginal productivity of capital equal to $\theta > 0$. We assume that in equilibrium both labor and capital are employed in the outside sector. Without loss of generality we choose the outside good to be the numeraire, which implies that the price of labor is 1 and the rental price of capital is θ.

The production of varieties of differentiated goods employs labor and capital according to a constant returns to scale Cobb-Douglas technology. One variety corresponds to one and only one product, which is supplied by one firm. For expositional clarity we assume that firms supply at most one product in the same market segment and are eventually active in multiple market segments.

The core competence of a firm is sufficient to develop its *core segment*. If the firm is active in other segments, additional know-how is necessary to customize a firm's operations, which implies that *non-core segments* are characterized by additional customization costs. The core segment of each firm is indexed with $i = 0$, the eventual non-core segments are indexed with $i = 1, 2, ..., m$ for a discrete number of segments m. Firms are heterogeneous in the core cost c and in the vector of customization costs $\boldsymbol{z} = \{z_i\}_{i=0}^m$. After entry, a firm learns about its core cost c as a random draw from an exogenous continuous distribution $G(c)$.

Since our goal is to model the behavior of firms whose organization is a conglomerate of unrelated segments, the model abstracts from vertical linkages between segments and economies of scope, such that production in each segment can be examined in isolation.

The production function in segment i with customization cost z_i of a firm with a marginal cost of the core competence c is given by:

$$y\left(z_i c\right) = \frac{\varphi}{z_i c} \, l\left(z_i c\right)^{\lambda} k\left(z_i c\right)^{1-\lambda}, \tag{2}$$

where $y\left(z_i c\right)$ is output, and $l\left(z_i c\right)$ and $k\left(z_i c\right)$ are labor and capital used in production, $\lambda \in (0,1)$ is the elasticity of output with respect to labor and the coefficient $\varphi = \left(\frac{1-\lambda}{\lambda\theta}\right)^{-(1-\lambda)} + \theta\left(\frac{\lambda\theta}{1-\lambda}\right)^{-\lambda}$ is a constant.

Firms maximise profits subject to the technology given by Equation (2). Profit maximization implies that the marginal cost in segment $i = 0, 1, 2, ...$ equals $z_i c$. Within a given firm we sort products by customization cost in increasing order, such that $z_0 = 1 < z_1 < z_2 < ... < z_m$. The equilibrium quantity, price, revenue, employment of labor, employment of capital, profit and return on assets $-$ measured as the ratio of profits to the cost of capital $-$ in a segment with marginal cost $z_i c$ are:

$$q\left(z_i c\right) = \frac{L}{2\gamma}\left(c_D - z_i c\right) \tag{3a}$$

$$p\left(z_i c\right) = \frac{1}{2}\left(c_D + z_i c\right) \tag{3b}$$

$$r\left(z_i c\right) = \frac{L}{4\gamma}\left(c_D^2 - \left(z_i c\right)^2\right) \tag{3c}$$

$$l\left(z_i c\right) = \frac{z_i c}{\varphi_l}\frac{L}{2\gamma}\left(c_D - z_i c\right) \tag{3d}$$

$$k\left(z_i c\right) = \frac{z_i c}{\varphi_k}\frac{L}{2\gamma}\left(c_D - z_i c\right) \tag{3e}$$

$$\pi\left(z_i c\right) = \frac{L}{4\gamma}\left(c_D - z_i c\right)^2 \tag{3f}$$

$$roa\left(z_i c\right) = \frac{\varphi_k}{2\theta}\left(\frac{c_D}{z_i c} - 1\right), \tag{3g}$$

where $\varphi_l = \left(\frac{\lambda\theta}{1-\lambda}\right)^{\lambda-1}\varphi$ and $\varphi_k = \left(\frac{\lambda\theta}{1-\lambda}\right)^{\lambda}\varphi$ are constants. The variable c_D is the maximum cost below which demand is positive in the firm's domestic market. We refer to c_D as the "cutoff cost". It is determined in equilibrium and will be a sufficient statistic to summarize the degree of competition in the output market, with a lower c_D representing tougher competition.

A direct implication of quadratic preferences (1) and linear technology (2) is that the markup, i.e. $p(z_i c) - z_i c$, and markup factor, i.e. $p(z_i c)/(z_i c)$, are decreasing in marginal cost $z_i c$ and increase with the cutoff cost c_D. Thus, the model features monopolistic competition in which variable markups are increasing in revenue. Under these circumstances the decentralized equilibrium is inefficient, in the sense that it does not replicate the unconstrained optimum: production at low-cost firms is too small; as pointed out in Nocco et al. (2014) and recently proven in a more general framework by Dhingra and Morrow (2019).

Return on assets $roa\,(z_i c)$ is proportional to the markup factor $p(z_i c)/(z_i c)$. As segments with lower marginal cost charge higher markups, they have a higher return on asset, despite belonging to the same firm. Heterogeneity in return on assets is the main driver of allocation through the internal capital market of a MSF, as we will discuss in the next section.

3 Multi-segment firms

In this section we add to the existing literature on flexible technology by modeling the organisation within a MSF. A MSF differs from a SSF in that the true cost of a non-core segment is known to the divisional manager only and it is not verifiable by the MSF's owner. The organization of the MSF firm and its internal capital is shaped by the following features:

(1) *Empire building managers* who strategically misreport the true cost of their divisions to maximize their private benefit from running bigger divisions;

(2) *Winner picking headquarters* that allocate capital by ranking managers according to their divisions' return on assets, which subjects managers to competition for funds.[4]

Actual realizations of marginal costs (of core and eventually non-core products) are not known before entry: decisions, both by firms and divisional managers, are based on their expected payoffs, given that the distribution of marginal costs is public knowledge.

[4]Empire Building managers have been introduced by Jensen (1986). The idea of an internal capital market disciplined by winner picking has been proposed by Stein (1997). Both arguments have since been tested extensively finding support in the data, see Stein (2003) for a review.

3.1 The decision to become a MSF

Firms endowed with core competence cost $c < c_D$, decide whether to become a MSF by opening a certain number of non-core divisions $m = 1, 2, ...$ or being a SSF that operates the core division only, i.e. $m = 0$. Opening a non-core division requires to pay a managerial compensation:

$$f_M + \eta\pi(z_i c), \tag{4}$$

where $f_M \geq 0$ is a fixed managerial wage and $\eta \in [0, 1]$ is a fraction of the profit of the division with marginal cost $z_i c$ that is paid to the divisional manager. Modeling the managerial cost as in Equation (4) accounts for the two components of executive's compensation which are typically designed to reach an optimal outcome of a principal (firm) agent (manager) problem within the organization: a fixed component captured by f_M and a performance-based component captured by $\eta\pi(z_i c)$.

Subtracting the cost of opening a division from the profit that a non-core division generates yields the value to the firm from the non-core division. The value is decreasing in the marginal cost of the division for every $\frac{z_i c}{c_D} \leq 1$, and positive for $z_i c \leq c_D - \sqrt{\frac{4\gamma f_M}{(1-\eta)L}} \equiv c_M(c_D)$, where we refer to $c_M(c_D) < c_D$ as the multi-segment costs cutoff. Only firms endowed with a core competence cost lower than the multi-segment cutoff, i.e.

$$c < c_D - \sqrt{\frac{4\gamma f_M}{(1-\eta)L}} \equiv c_M(c_D), \tag{5}$$

have the incentive to become a MSF.

Opening a division requires hiring divisional managers. In what follows, we first describe managers' behavior once they are in charge of a division; then, we model how a MSF "picks" managers for a given number of divisions out of a pool of potential candidates.

3.2 Empire building managers

A divisional manager has private knowledge on the true customization cost of her own division, which we denote by $x_i \geq 1$ and distinguish from $z_i \geq 1$ that is the customization cost reported to the firm. We assume an empire building behavior, meaning that the manager extracts a non-pecuniary and non-verifiable private benefit from running a bigger division. In this context, the manager has the incentive to mis-report costs, i.e. $z_i \neq x_i$, when this leads to attract more capital $k(z_i c) > k(x_i c)$ than by disclosing the true customization cost. Under these circumstances, the division operates at the reported marginal cost, i.e. $z_i c$, with performances (3d)–(3e) that are observable to the firm and that the manager commits to, but they differ from performances that would have been possible at the true marginal cost $x_i c$.

Thus, the division supplies an output of $q(z_i c) = y(z_i c)$ units, while the maximum output obtainable running the division at the true marginal cost is $\frac{z_i}{x_i} y(z_i c) > y(z_i c)$. In line with the literature, we assume that the divisional manager acts to maximize a private benefit measured by the excess of output capacity:[5]

Definition. *Let $\mu = z_i/x_i$ be the factor of mis-reporting. A manager's non-pecuniary private benefit equals excess output capacity*

$$b\left(\mu; x_i, c, c_D\right) :\equiv \frac{z_i}{x_i} y(z_i c) - q(z_i c) = \frac{L}{2\gamma}\left(\mu - 1\right)\left(c_D - \mu x_i c\right), \tag{6}$$

conditional on $k(z_i c) > k(x_i c)$ and it is zero otherwise.

This definition of managers' private benefits implies that (i) managers gain a private benefit only if they obtain an excess capital allocation $k(z_i c) > k(x_i c)$ without under-reporting their effective cost, i.e. $\mu \geq 1$; (ii) the private benefit increases in over-reporting, i.e. in μ; (iii) managers enjoy greater private benefits the better their know-how (lower x_i) and the better the firm they match with (lower c); and (iv) tougher competition, i.e. a lower cutoff c_D, decreases the private benefit.

3.3 Winner picking

A MSF opening $m = 1, 2, ...$ non-core divisions announces a tournament to hire as many divisional managers. Given the continuum of households, the pool of potential managers is unrestricted, each endowed with an idiosyncratic level of know-how that make her

[5] Scharfstein and Stein (2000) model the private benefit of empire building managers as an excess of output capacity. In appendix A Section 7.1 we show that all the results are robust to a more general definition of private benefit, as represented by any continuous and twice differentiable function $b\left(\mu; x_i, c, c_D\right)$ that satisfies a minimal set of sufficient conditions.

capable of running a division with a certain customization cost. Individual realizations are not observable, but firms and managers have public knowledge on the exogenous distribution of customization costs at which divisions can be operated.[6]

Before applying, managers know the number of divisions that a firm is opening, the firms' core marginal cost and the toughness of market competition.[7] The rules of the tournament are:

1. The firm meets once, randomly, with an unrestricted number of candidates, at no cost (i.e. *free entry* into the tournament). Independently and simultaneously, each candidate proposes a customization cost z_i at which she would run a division.

2. It is understood by both parties that, in case an agreement is reached, the division will perform according to the equilibrium allocation (3a)-(3g), given the proposed customization cost. These performances are verified by the firm ex-post, and punishment for missing these targets is prohibitive.

3. The firm ranks managers by return on asset given the proposed customization cost $roa(z_i c)$ and commits to finance the first m managers, with the corresponding capital allocation $k(z_i c)$. Managers who are not chosen by any MSF joint the pool of employees (i.e. managers' *outside option* is earning the same wage).

This simple design is sufficient to characterize three key implications. First, only managers who propose a customization cost $z \leq c_M(c_D)/c \equiv \bar{z}(c, c_D)$ can lead a profitable division at a firm with core marginal cost c when the cutoff cost is c_D. It follows that the distribution of customization costs among applicants to the same firm is endogenous, since it is a truncation of the exogenous distribution below a $\bar{z}(c, c_D)$.

Second, the sole goal of candidate managers is to be hired, i.e. to be ranked among the best m applicants, since all other aspects of their appointment (such as resource allocation and targets) are fixed. Therefore, within the pool of applications, each candidate competes against arbitrarily many other candidate managers, while taking the

[6]Note that assuming an exogenous distribution of customization costs, i.e. z, and an exogenous distribution of core competence costs, i.e. c, as we will when closing the model, is equivalent to assume that firms face an exogenous distribution of marginal costs, i.e. zc. This is standard in the literature on heterogeneous firms making forward looking decisions under free entry.

[7]When considering how to supply their time as employees or managers, we assume that households view private benefits of running a division (6) as prestige (a non pecuniary dimension), and that they have lexicographic preferences between utility from consumption and prestige. Therefore, households look for maximizing utility from consumption first, and then prestige matters only given the same utility from consumption.

firm-specific distribution of reported customization costs among them as given. The consequence is that the customization costs reported by managers who win the tournament are random draws from the firm-specific conditional distribution of the joint event $\{z_i \leq \bar{z}(c, c_D) \forall i = 1, ..., m\}$.

Third, given free entry into the tournament and common managers' outside option, on the one hand, the firm cannot gain by providing incentives to managers, since the firm's expected profit from non-core divisions depends on the firm-specific distribution of applications ex-ante. On the other hand, managers have no bargaining power vis-à-vis the firm. Therefore, it is optimal and feasible for a MSF not to offer any profit share, i.e. $\eta = 0$, and to pay managers the value of their outside option, i.e. $f_M = w = 1$.

In conclusion, competition among managers in the tournament determines an equilibrium in which MSFs maximise expected profit subject to the exogenous distribution of marginal costs.

Each MSF runs the tournament by announcing a number of divisional manager positions. But how many? A trade-off emerges between opening many but on average worse non-core divisions, or opening few but on average more profitable non-core divisions.[8] However, withou economies of scope at the level of segments the two forces cancel out such that the optimal number of non-core divisions is undetermined (i.e. the expected profit from the pool of non-core divisions does not depend on the number of non-core divisions). This insight is sufficient to characterize the probability that an application for a managerial position is financed:

Lemma. *Without economies of scope, the probability that a manager offering a customization cost* $z_i = \mu x_i < \bar{z}(c, c_D)$ *is financed is given by:*

$$\psi(\mu; x_i, c, c_D, m) = 1 - \frac{F(\mu x_i)}{mF(\bar{z}(c, c_D))} \tag{7}$$

for a given exogenous function $F : [1, z_{max}] \rightarrow [0, 1]$ *that is continuous and increasing, where the parameter* $z_{max} > 1$ *is an arbitrary finite upper bound of the support. Under*

[8] On the one hand, fewer non-core divisions imply lower customization costs (because a candidate manager has a lower probability to be in the top-m when offering a higher customization cost) and thus a higher expected profit per division. On the other hand, the expected total profit grows with the number of non-core divisions.

the assumption that $F(z)^{\frac{1}{m}}$ is convex on $[1, z_{max}]$ the following comparative statics hold:[9]

$$\frac{\partial\psi}{\partial\mu} < 0, \quad \frac{\partial\psi}{\partial x_i} < 0, \quad \frac{\partial\psi}{\partial c} < 0, \quad \frac{\partial\psi}{\partial m} > 0, \quad \frac{\partial\psi}{\partial c_D} > 0 \ ;$$
$$\frac{\partial\psi^2}{\partial\mu^2} \leq 0, \quad \frac{\partial\psi}{\partial\mu\partial x_i} < 0, \quad \frac{\partial^2\psi}{\partial\mu\partial c} < 0, \quad \frac{\partial^2\psi}{\partial\mu\partial m} > 0, \quad \frac{\partial^2\psi}{\partial\mu\partial c_D} > 0 \ .$$

Proof. See appendix A Section 7.2.

3.4 Excess capital allocation

Empire building managers choose the factor of mis-reporting that maximises their expected private benefit:

$$\mu_i^\star = \arg\max_{\mu\geq 1} \ \psi\left(\mu; x_i, c, c_D, m\right) b\left(\mu; x_i, c, c_D\right) \ , \tag{8}$$

The manager's problem (8) has a unique solution, which is characterized by the following properties:

Proposition 1. *Better managers, i.e. those characterized by $1 \leq x_i < \frac{c_D}{2c}$, over-report the customization cost $\mu_i^\star(x_i, c, m, c_D) > 1$, and the comparative statics describing their optimal decision are such that:*
(1.1) $\mu_i^\star$ *is decreasing in the true customization cost of the division x_i,*
(1.2) $\mu_i^\star$ *is decreasing in the core competence cost of the firm c,*
(1.3) $\mu_i^\star$ *is increasing in the number of non-core divisions, m*
(1.4) $\mu_i^\star$ *is increasing in the market cutoff cost c_D.*

Instead, worse managers, i.e. those characterized by $\frac{c_D}{2c} \leq x_i < \frac{c_M(c_D)}{c}$, report their actual customization cost x_i.

Proof. See appendix A Section 7.3.

Candidate managers who report a customization cost $z_i > \frac{c_D}{2c}$ reveal themselves as "bad managers" types. This follows from the the fact that, for a true customization cost of

[9] Convexity is only a sufficient condition. In the proof of the Lemma (in the appendix) we show that these comparative statics hold under a less restrictive necessary condition.

$x_i < \frac{c_D}{2c}$, a rational manager is strictly better off by reporting $z_i \leq \frac{c_D}{2c}$: she attains both a greater payoff and a greater probability of being financed. Candidate managers who report a customization cost $z_i \leq \frac{c_D}{2c}$ thus reveal themselves to be of the "good managers" type. Otherwise, by under-reporting their effective cost, they would commit to performances (see Equations (3a)–(3g)) that they cannot deliver, which would lead to a prohibitive punishment.

The first order condition of the manager's problem (8) can be written as the semi-elasticity of managers' payoff $b(\mu)$ with respect to the over-reporting factor $\xi_{pay}(\mu)$ equal to the semi-elasticity of the probability of being financed $\psi(\mu)$ with respect to the over-reporting factor $\xi_\psi(\mu)$. The definition of the private benefit (6) and the properties of the probability of being financed (7) are sufficient to prove that $\xi_{pay}(\mu)$ is decreasing in μ, while $\xi_\psi(\mu)$ is increasing in μ, and $\xi_{pay}(1) > \xi_\psi(1)$.[10] The semi-elasticities $\xi_{pay}(\mu)$ and $\xi_\psi(\mu)$ correspond to two curves that cross in a unique point in the plan drawn over the support $\mu > 1$. We refer to $\xi_{pay}(\mu)$ as the curve describing the *marginal gain in the expected payoff*, and we refer to $\xi_\psi(\mu)$ as the curve describing the *marginal loss in the probability of being financed*. The driving forces behind the comparative statics in Proposition 1 are shown in Figure 2.

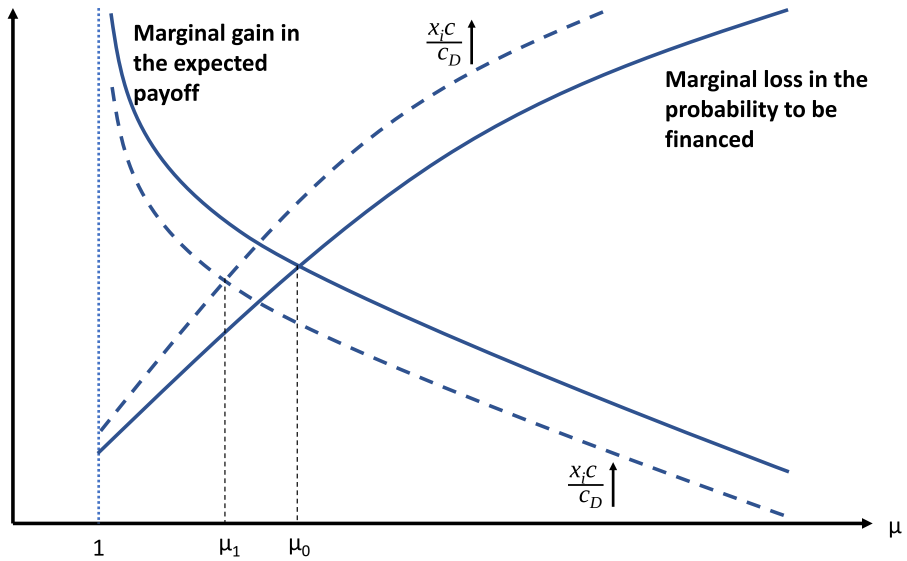
Panel 2a summarizes results (1.1), (1.2) and (1.4) from Proposition 1. The second order derivatives in Equation (6) show that the gains in terms of private benefits due to over-reporting decline in the effective marginal cost $x_i c$ and increase in the cutoff cost c_D. Therefore, a higher marginal cost relative to the market cutoff pushes down the curve describing the marginal gain in the expected payoff. The second derivative of the probability of being financed in Equation (7) shows that managers of divisions with higher marginal costs relative to the cutoff cost have less room to over-report, and that their likelihood to be financed declines as they over-report more. Thus, the decline in the probability of being financed is larger the greater the ratio $\frac{x_i c}{c_D}$ – the corresponding curve moves up. In sum, the optimal level of over-reporting is lower when the true customization cost of a division and/or the firm core competence cost are higher relative to the cutoff cost.

Panel 2b describes result (1.3) from Proposition 1. Over-reporting is more pronounced in firms with a greater number of non-core divisions. As more positions need to be filled, it is more likely that a manager's offer will be in the range of selected offers, all else equal. The manager thus has less to lose by over-reporting: the curve describing the change in the

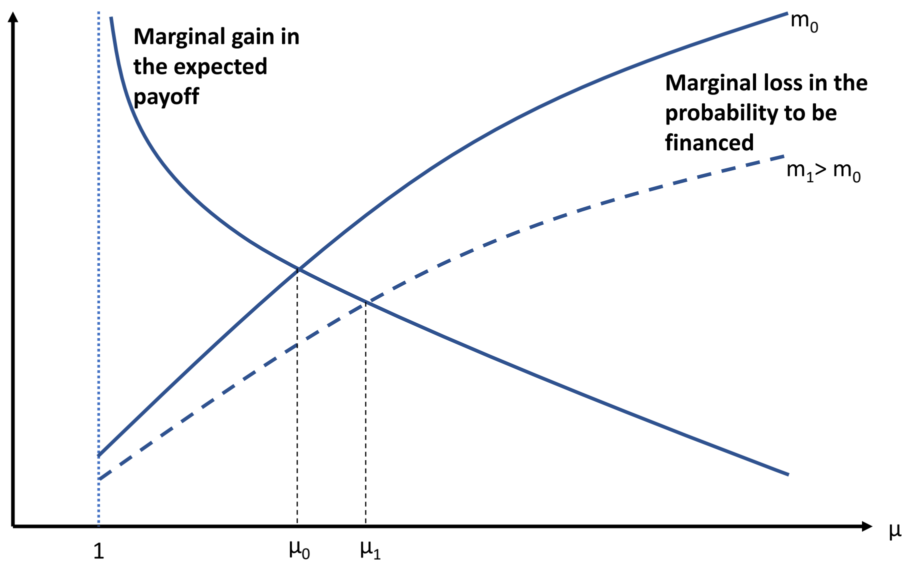
[10] We prove these results in Lemma 1.1 - 1.4 of the proof of Proposition 1, in the appendix.

Figure 2: **Comparative statics on the optimal factor of over-reporting**

(a) Over-reporting is lower the higher the effective marginal cost $x_i c$ relative to the cutoff cost c_D



(b) A greater number of non-core divisions m increases over-reporting



probability of being financed shifts down as the number of non-core divisions increases. The expected payoff is not affected by the number of divisions. Therefore, managers' optimal level of over-reporting is higher the greater the number of non-core divisions.[11]

The analysis on capital allocation across segments within MSFs has four implications. First, managers running a division with relatively low marginal costs have more room for over-reporting, while still facing a relatively high probability of being financed. Therefore, within a given firm, managers of better divisions over-report their costs relatively more. Across firms, over-reporting is larger in better firms (with lower core marginal cost).

Second, over-reporting is lower if the market is more competitive, i.e. the lower the cutoff cost c_D. A lower market cutoff reduces the range of costs at which a division is financed. Managers internalize this channel when they over-report the cost of their division. The toughness of competition in the output market thus acts as a disciplining device on managers' strategic over-reporting. Through this mechanism, which is distinctive to our theory, the marginal cost of a non-core division in a MSF responds endogenously to external changes in competition. Therefore, the model yields direct implications on how competition affects resource allocation within MSF, and these implications go beyond a simple reallocation of output market shares.

Third, the allocation of capital across segments (given by Equation (3e)) follows an inverted-U shaped function of marginal costs at the segment level. For better managers (i.e. those with $z_i c < c_D/2$), allocated capital increases with marginal costs, but eventually declines as managers get worse and marginal costs increase (for those with $z_i c \geq c_D/2$).

Fourth, the degree of competition affects the allocation of capital within firms by changing managers' strategic choice of over-reporting. The asymmetric over-reporting of customization costs (which is more pronounced the lower a segments' true marginal cost) implies that, within the same MSF, the better non-core divisions receive more capital than under true reporting; and across firms, better MSFs allocate too much capital to their best non-core divisions, relative to a world of no over-reporting.

In conclusion, Proposition 1 describes managers' strategic behavior, given the true marginal cost $x_i c$, market cutoff cost c_D, and the number of non-core divisions at the firm m. Changes in the degree of over-reporting thus affect the allocation of capital within MSFs.

[11] In the appendix we show that result (1.3) in Proposition 1 is supported by the data: average marginal costs are higher among firms with a greater number of non-core segments.

3.5 Endogenous marginal cost

In the recent literature on multi-product firms, competition in the output market lowers the average cost in the economy by reallocating market shares from high-cost firms to low-cost firms and from high-cost products to low-cost ones, acting both at the extensive (selection) and at the intensive margins.[12] These channels are common to our model as well, but our theory delivers a novel mechanism on how competition affects resource allocation within MSFs which goes beyond a reallocation of output market shares. In this section we show how the introduction of an internal capital market allows us to endogenise the cost structure of multi-segment firms, which remains exogenous in the multi-product-firms literature.

Managers strategically over-report the cost of their divisions, but they do so in a way that depends on the true customization cost of the division x_i, on the core marginal cost of the firm c, on the number of non-core divisions m and on competition in the output market, through c_D. Therefore, through managers' strategic over-reporting, the model generates endogenous heterogeneity in the cost structure between segments within firms (differences in customization costs); between firms (differences in core competence costs); between firms' organization (differences in the number of divisions); and in response to competition. Importantly, our model highlights a new margin through which market competition and international trade impacts MSFs. Affecting managerial over-reporting differently across divisions, competition moves endogenously the full structure of costs between segments, an effect that is absent when these marginal costs are exogenous.

To illustrate this point we compare the relative cost across divisions in our theory with the one in Mayer et al. (2014) which is a setup analogous to ours but without an internal capital market and strategic over-reporting of costs. In their model the relative marginal cost between two products of the same firm, say i and j, is exogenously fixed by the ratio in their customization costs. In our model the relative marginal cost between two non-core products becomes endogenous to the four determinants of over-reporting, as shown in the following equation

$$\frac{x_i}{x_j} \neq \frac{\mu^\star(x_i, c, m, c_D)}{\mu^\star(x_j, c, m, c_D)} \frac{x_i}{x_j} \ ,$$

where the strategic over-reporting by two managers generates a distortion to the allocation

[12]See Mayer et al. (2014) and results that go in the same direction are common in many models of multi-product firms, see Hopenhayn (2014).

of capital toward more financing for the better products, within the same firm.

Similarly, the marginal cost of two non-core products with the same customization cost but produced by different firms, say with core marginal costs c' and c'',

$$\frac{c'}{c''} \neq \frac{\mu^\star(x_i, c', m, c_D)}{\mu^\star(x_i, c'', m, c_D)} \frac{c'}{c''} \ ,$$

are no longer exogenously given as in Mayer et al. (2014), but depend on the four factors affecting mis-reporting $\mu(x_i, c, m, c_D)$. Our model predicts that more capital is allocated to the product of the better firm.

4 Free entry equilibrium in an open economy

Entry in the differentiated good sector is costly. Entrants undertake an irreversible investment of $f_E > 0$ units of labor through which they learn about their core marginal cost $c \sim G(c; \rho)$, where the parameter ρ accounts for the concentration in the distribution of core marginal costs.

Firms that are not profitable exit at zero value. Firms with $c \leq c_D$ produce at least in their core segment, making a profit $\pi(c)$. Among incumbent firms, those that satisfy the necessary condition (5), i.e. $c \leq c_M(c_D)$, look for managers and become MSFs with probability $F(\bar{z}(c, c_D))$. Within the group of MSFs, non-core divisions with customization cost $z \leq \bar{z}(c, c_D)$ are financed in a firm with core competence cost c, for a given cutoff cost of c_D. In a free entry equilibrium with a positive mass of firm entrants the expected value before entry must be equal to the sunk cost of entry:

$$\begin{aligned} \Pi(c_D) &= \int_0^{c_D} \pi(c) dG(c; \rho) \\ &+ \int_0^{c_M(c_D)} \left(\int_1^{\bar{z}(c, c_D)} [\pi(zc) - f_M] dF(z) \right) dG(c; \rho) = f_E \ . \end{aligned} \tag{9}$$

The free entry condition (9) determines the cutoff cost c_D. Every exogenous shock leading to more competition, i.e. a lower cutoff cost c_D, has general equilibrium effects at the extensive margin, on the set of both firms and segments, and on the intensive margin. We focus on the channel of international trade as the source of changes in competition.

4.1 Open economy

In an open economy setting, firms sell in their domestic market, but might also export to a foreign market. For the sake of exposition we assume that the foreign market is symmetric to the domestic one, that product markets are segmented, and that exported products are subject to a per-unit trade cost $\tau \geq 1$.[13]

Under these circumstances, it can be shown that a domestic producer with core marginal cost c would serve the foreign market as a local producer with core marginal cost τc. Firms export to a foreign market only in segments in which they make a positive profit. Thus, only firms with a core competence cost $c \leq c_D/\tau$ become exporters, at least for their core segment. Among MSFs, only non-core segments with a marginal cost $z_i c \leq c_D/\tau$ have positive sales in the foreign market.

Expansion into a foreign market and import competition affect the expected value of a firm at entry into the domestic market. The free entry condition (Equation (9)) evaluated in the open economy setting determines the cutoff cost for entry in the domestic market c_D as an increasing function of the trade cost τ. Thus, an exogenous reduction in trade costs (e.g. a trade liberalization policy) is responsible for an tougher competition in the output market, that can be measured as a reduction in the cutoff cost c_D.[14]

4.2 Tobin's Q and the conglomerate discount

In the previous sections we showed how competition and international trade affect the capital allocation across segments. In this section we link the capital allocation across segments to the conglomerate discount.

Total profits and capital of a firm with core marginal cost c and customization costs $\boldsymbol{z} = \{z_0, z_1, ..., z_m\}$ are given by $\pi_{tot}(c) = \sum_{i=0}^{m} \left[\pi\left(z_i c\right) + \mathbf{1}_{[z_i c]} \pi\left(z_i \tau c\right)\right]$ and $k_{tot}(c) = \sum_{i=0}^{m} \left[k\left(z_i c\right) + \mathbf{1}_{[z_i c]} k\left(z_i \tau c\right)\right]$ respectively; where $\mathbf{1}_{[z_i c]}$ is an indicator function that is $\mathbf{1}_{[z_i c]} = 1$ if there are exports in segment i of a firm c and zero otherwise. We refer to the value of total firm capital $\theta k_{tot}\left(\boldsymbol{z}, c\right)$ as "*book value*", and the discounted lifetime stream of profit $\frac{1+\theta}{\theta} \pi_{tot}\left(\boldsymbol{z}, c\right)$ as "*market value*" of the firm. The market-to-book ratio, or *Tobin's*

[13] Mayer et al. (2014) show how the same framework can be extended to many countries with asymmetric characteristics and bilateral trade costs.

[14] In appendix C (available online) we derive the open economy solution of the model, and we dedicate Section 9.1 to a detailed discussion of comparative statics about the toughness of market competition.

Q is given by:

$$T(\boldsymbol{z}, c) = \frac{(1+\theta)\varphi_k}{2\theta^2} \frac{c_D^2 - 2c_D \mathbb{A}[\boldsymbol{z}c] + \mathbb{A}[(\boldsymbol{z}c)^2]}{c_D \mathbb{A}[\boldsymbol{z}c] - \mathbb{A}[(\boldsymbol{z}c)^2]}. \tag{10}$$

where $\mathbb{A}[\boldsymbol{z}c] = [1 + m + \sum_{i=0}^{m} \mathbf{1}_{[z_i c]}]^{-1} \sum_{i=0}^{m} (z_i + \mathbf{1}_{[z_i c]} z_i \tau) c$ and $\mathbb{A}[(\boldsymbol{z}c)^2] = [(1+m) + \sum_{i=0}^{m} \mathbf{1}_{[z_i c]}]^{-1} \sum_{i=0}^{m} (z_i^2 + \mathbf{1}_{[z_i c]} z_i^2 \tau^2) c^2$ are, respectively, the first and second moment of the distribution of marginal cost across divisions of the firm. Tobin's Q decreases with average marginal cost $\mathbb{A}[\boldsymbol{z}c]$ and, conditional on the average, increases with the variance of marginal costs across divisions $\mathbb{A}[(\boldsymbol{z}c)^2] - \mathbb{A}[\boldsymbol{z}c]^2$.

Over-reporting affects Tobin's Q, and hence the conglomerate discount, by changing average marginal costs and their dispersion. First, it directly increases average marginal cost $\mathbb{A}[\boldsymbol{z}c]$. Second, holding average marginal costs fixed, the relatively more pronounced over-reporting at better divisions decreases the dispersion of marginal costs $\mathbb{A}[(\boldsymbol{z}c)^2] - \mathbb{A}[\boldsymbol{z}c]^2$, because costs of better divisions move closer to costs of worse divisions. Intuitively, the closer divisions are in their costs, the less valuable is the winner picking role of the internal capital market. Both channels decrease the Tobin's Q of MSFs. SSFs are not subject to over-reporting, as there are no informational frictions between the owner and any divisional manger.

Therefore, although the core marginal cost of a MSF (c_{msf}) is lower than the core marginal cost of a SSF (c_{ssf}), the model accounts for the possibility of a "*conglomerate discount*", such that $T(\boldsymbol{z}, c_{msf}) / T(1, c_{ssf}) < 1$. A conglomerate discount arises if the difference in core marginal costs between single- and multi-segment firms ($c_{ssf} - c_{msf} > 0$) is sufficiently small relative to the cost structure of non-core segments within the MSF. The model hence shows that the over-allocation of capital to better divisions of a MSF increases the average and decreases the dispersion of marginal costs across segments within the MSF. Both channels inflate the cost structure of non-core divisions, and more so the greater strategic over-reporting by divisional managers. Therefore, a conglomerate discount reflects mis-allocation in the internal capital market of MSFs.

The mode suggests that productive MSFs may exhibit a smaller conglomerate discount despite a distorted allocation of capital. The reason is that, on average, their core marginal cost is lower. In the most productive multi-segment firms exposed to the disciplining effect of international trade, the conglomerate discount may disappear or even become a premium (see Figure 1).

5 Taking the model to the data

This section derives a set of predictions from the model and then tests them with US firm- and segment-level data. In line with these predictions, we show that informational frictions distort the allocation of capital across segments within multi-segment firms; and that competition has a disciplining effect on firms' internal capital allocation.

5.1 Data and main variables

Compustat provides yearly balance sheet information on publicly listed US firms. The sample period spans 1999 to 2016. In addition, we observe sales, assets, profits, and return on assets at the 4-digit SIC segment level. We follow the literature and drop observations for which the sum of reported segment sales does not fall within 25% of total firm sales. We further eliminate segments with the name 'other' or a SIC classification of 0, or missing and anomalous accounting data (eg missing sales, assets, capital expenditure, depreciation, operating profits; or zero depreciation, capital spending greater than sales or assets, capital spending less than zero). Finally, we eliminate firms and segments that operate in regulated industries (see eg Ozbas and Scharfstein (2010)).[15] Importantly, we follow Ozbas and Scharfstein (2010) and focus on unrelated segments. To this end, we use BEA data on input-output relationships among industries and exclude all vertically related industries (an industry that buys/sells more than 10% of its inputs/outputs from/to the other industry). Firms in the sample operate in up to ten industries (segments). All firms that have only one segment are classified as single-segment.

Central to the analysis is the segment-level return on assets, which we use to derive segments' marginal costs relative to the market cutoff ($z_i c / c_D$ in the model). As is standard, we compute segment profits as operating income plus depreciation; and we compute return on assets as profit over asset.

Calculating profits from income statements can lead to a biased measure of actual profits. For example, in the presence of adjustment frictions, the accounting measure does not adequately reflect the actual expenditure on capital (De Loecker et al., 2020). We thus adjust segment-level return on assets as follows: First, we follow De Loecker et al. (2020) to compute firms' profit rates from firms' balance sheet items.[16] Balance

[15] These cover SIC codes 4000–4999, 6000-6199, and 6300-6499.

[16] The profit rate is calculated as total sales minus all costs (including overhead and the expenditure

sheet information required to compute the profit rate is only available at the HQ level. In a second step we thus adjust each segment's return on asset with the respective ratio between the profit rate and return on assets at the HQ level.[17] Adjusting the segment-level return on assets in this way alleviates concerns that could arise if we were to compute return on assets solely based on segments' income statements.

For our analysis, we use a measure of marginal costs derived from return on assets. In particular, Equation 3c shows that relative marginal costs are an inverse function of the return on assets (RoA), which we observe at the segment level. Theory expresses variables in terms of marginal cost relative to the cutoff value, $z_{fst}c/c_{Dt}$. Rearranging, the model yields $rmc_{fst} = 1/(1 + \frac{2\theta}{\varphi_k} returns_{fst})$. Here, θ is the rental price of capital on the external market and φ_k is the capital share of total cost, both of which are the same across firms and segments. We thus calculate a model-driven measure of relative marginal costs, based on the adjusted segment RoA, as

$$\widetilde{rmc}_{fst} = \frac{1}{1 + RoA_{fst}}. \tag{11}$$

However, marginal costs $\widetilde{rmc}_{fst}$ might be in part determined by other potentially confounding factors. We thus purge our measure of marginal costs from confounding factors as follows: We regress the measure of segment-level relative marginal costs computed from the (adjusted) segment-level return on assets on the yearly average interest rate, average capital to labor ratio, average concentration (Herfindahl-Hirschman index, HHI) and average sales growth in each segments' industry. In addition, we factor out common trends through year fixed effects, as well as differences across firm size through dummies for quartiles of the employment distribution. Firm size dummies also partly control for differences in technology across firms.[18] The residual from this regression is our measure

on capital) as a share of sales. Crucially, this measure of the profit share differs from the accounting-based computation of profits, as it uses a measure of capital obtained from the balance sheet and not the income statement (see De Loecker et al. (2020)).

[17] Specifically, $roa_s^{adjusted} = roa_s \times \frac{profit\ rate_f}{roa_f}$. The underlying assumption is that the observed discrepancy between the profit rate and return on assets at the HQ-level also occurs at the segment level.

[18] We estimate $rmc_{fst} = \alpha + \delta_1 interest_{st} + \delta_2 (k/l)_{st} + \delta_3 HHI_{st} + \delta_4 \Delta sales_{st} + \delta_5 quartiles_f + \tau_t + \varepsilon_{fst}$, where f denotes firm, s segment and t time. The introduction of sectoral interest rates, concentration and sales growth captures changes in the competitive environment due to market access, aggregate supply and aggregate demand channels. The sectoral capital to labor ratio, firm size dummies and year fixed effects account for common trends in technical change and differential effects due to economies of scale. All measures are computed from Compustat data. The interest rate is the average interest expense over total debt, the capital labor ratio is total fixed assets to employment, and the HHI is based on total

of segment marginal costs, which we denote as rmc_{fst}. Note that all our results are qualitatively similar when we use unadjusted marginal costs in the analysis.

We further collect information on CEO tenure and the number of boards from Execucomp and BoardEX. Specifically, for each firm we compute CEO tenure as 2013 minus the year the average CEO joined the company, and compute the total number of boards the average CEO sits on. We use data on CEO tenure and the number of boards as a proxy for informational frictions. CEOs that have served for only a short period are expected to know their segments less well than CEOs that have already spent several years in the company.. Similarly, CEOs serving on boards of several companies are likely less familiar with individual segments of these companies (Eppler and Mengis, 2004). We define a firm as *high friction*, i.e. subject to high informational frictions, if CEO tenure is below the sample median or the CEO sits on a number of boards above the sample median.

Finally, to measure import competition at the 4-digit SIC industry level, we use the growth of Chinese import penetration by industry from 1999–2007 (the 'China shock') from Autor et al. (2013). As import penetration could be driven by unobservable industry factors, we collect data on imports from China to eight other advanced economies.[19] As Autor et al. (2013) show, the instrument isolates the supply component in observed imports (i.e. the variation in imports that is due to rising productivity in China and not due to observable or unobservable changes in the US economy). On aggregate, Chinese import penetration increased from 0.6% to 4.6% between 1991 and 2007.

Our final sample covers 4,202 individual firms and 7,264 individual segments. Around one-fifth of firms are multi-segment firms. The median multi-segment firm has 3 segments, with a maximum of 11. The unconditional conglomerate discount equals 16.4%; in other words, multi-segment firms' Q is over 16% lower than that of single-segment firms. Data on imports from China is only available for firms in the manufacturing sector. In our firm-level analysis on competition we thus need to restrict the sample to manufacturing

sales.

[19]Autor et al. (2013) define the growth of Chinese import penetration for industry s from 1999–2007 as $\Delta China_i = \Delta M_i/(Y_{i0}+M_{i0}-X_{i0})$, where ΔM_i is the growth in US imports from China from 1999–2007, which is divided by initial absorption (US industry shipments plus net imports, $Y_{i0}+M_{i0}-X_{i0}$) in the base period 1991, which is near the start of China's export boom. The authors argue that rising imports from China reflect a supply shock. China's falling prices, rising quality, and diminishing trade and tariff costs in these surging sectors are causes of its manufacturing export growth. The eight other advanced economies Australia, Denmark, Finland, Germany, Japan, New Zealand, Spain, and Switzerland. See Autor et al. (2013) for details.

firms active between 1999 and 2007. The online appendix provides descriptive statistics.

5.2 Deriving testable implications

Combining the equations for capital allocation (3e) and equilibrium cutoff costs with the definition of reported marginal cost ($z_{fs}c_f = \mu_{fs}x_{fs}c_f$) yields the core set of equations at the segment level:

$$ln\left(c_{Dt}\right) = \frac{1}{2+\rho}\left[ln\left(\frac{\gamma\phi f_e}{\Omega L}\right) - ln\left(1+\tau_t^{-\rho}\right)\right], \tag{12a}$$

$$ln(assets_{fst}) = ln\left[rmc_{fst}(1-rmc_{fst})\right] + 2ln\left(c_{Dt}\right) + ln\left(\frac{L}{2\gamma\varphi_k}\right). \tag{12b}$$

The variable rmc_{fst} denotes marginal costs relative to the market cutoff of segment s in firm f in year t. Equation 12a shows that a fall in trade costs implies a lower market cutoff cost. Equation 12b shows how segment assets depend on the market cutoff and relative marginal costs.

Dynamics in the model are driven by three time-varying variables: the trade cost (τ_t), the market cutoff cost (c_{Dt}), and the over-reporting factors (μ_{fst}). The key channel is that falling trade costs, by lowering the cutoff value, lead to a decline in the over-reporting factor: under pressure of tougher competition, managers reduce the extent to which they over-report the cost of their division. In consequence, the allocation of assets across segments changes, as it depends on relative marginal costs.[20] To the extent that our analysis exploits shocks at the industry level (i.e., the China shock), common factors that affect firms within an industry will not invalidate our estimation – we will show the robustness of our findings to accounting for common factors in more detail below.

In what follows, we will refer to *marginal costs relative to the market cutoff* as either *relative marginal costs* or simply *marginal costs.* We now derive testable predictions from the model based on equations (12a)–(12b).

[20] Additional factors capturing market size (f_E, L), preferences (γ), and technology (Ω, ϕ, ρ, φ_k and $x_{fs}c_f$) are constant over time. In our estimation, we include fixed effects to control for these (unobservable) variables.

5.2.1 Capital allocation and informational frictions

In the model, an inefficient allocation of capital within multi-segment firms arises from segment managers' desire for empire building and over-reporting, which is enabled by informational frictions: as the headquarters cannot verify the true marginal costs of its divisions, managers can over-report marginal costs. Theory thus predicts that the problem of capital over-allocation is worse in firms in which divisional managers have more room for over-reporting, i.e., in firms with more severe informational frictions. The fact that there is no such incentive conflict in single-segment firms leads to the following predictions:

Prediction 1.1 *Within-firm capital allocation: Multi-segment firms allocate more capital to their non-core segments than single-segment firms with similar marginal costs.*

Prediction 1.2 *Informational frictions: The degree to which multi-segment firms over-allocate capital to non-core segments, relative to single-segment firms with similar marginal costs, increases in informational asymmetries.*

To test Prediction 1.1, we estimate the following regression equation at the segment-year level:

$$\begin{aligned} log(assets)_{fst} =& \gamma_1 \ multi\ segment_f + \gamma_2 \ rmc_{fst} \\ & + \gamma_3 \ multi\ segment_f \times rmc_{fst} + \zeta_f + \iota_i + \epsilon_{fst}, \end{aligned} \tag{13}$$

where $log(assets)$ is capital allocated to segment s of firm f in year t, $multi\ segment_f$ denotes a dummy that takes on a value of one if a segment is part of a multi-segment firm, and rmc_{fs} denotes the marginal cost of a firm's segment. For single-segment firms, the core segment is equivalent to the firm. To control for constant factors affecting asset allocation across firms in the same sector ($ln\left(\frac{L}{2\gamma\varphi_k}\right)$ in Equation 12b), we include industry fixed effects ι_i. We further control for firm size through dummies for quartiles of the firm employment distribution (ζ_f); in other words, our regressions compare multi- and single-segment firms of similar sizes in the same industry. Standard errors are clustered at the firm level. We expect that, conditional on the same marginal cost, multi-segment firms allocate more assets to a segment than single-segment firms ($\gamma_3 > 0$).

Note that marginal costs might be subject to measurement error. However, to the extent that measurement error is uncorrelated with the status of firms as single-segment or multi-segment firms, such measurement error will lead to attenuation bias.

Table 1: **Within-firm capital allocation and informational frictions**

VARIABLES	(1) log(assets)	(2) log(assets)	(3) log(assets)	(4) CEO nr boards log(assets)	(5) CEO time log(assets)
multi-segment	-1.754***	-1.835***	-2.005***	-0.993**	-0.792
	(0.402)	(0.403)	(0.458)	(0.430)	(0.529)
marginal costs	-1.023***	-1.095***	-1.098***	-0.328	-0.729*
	(0.291)	(0.294)	(0.323)	(0.395)	(0.397)
multi-segment × marginal costs	0.885**	0.973**	1.173**	-0.040	-0.236
	(0.431)	(0.433)	(0.492)	(0.471)	(0.570)
high friction				0.610	0.733
				(0.405)	(0.516)
multi-segment × high friction				-1.458***	-2.582***
				(0.536)	(0.742)
marginal costs × high friction				-0.923**	-0.825
				(0.443)	(0.543)
multi-segment × marginal costs × high friction				1.785***	2.996***
				(0.587)	(0.796)
Observations	16,982	16,982	16,982	16,982	16,982
R-squared	0.531	0.538	0.599	0.601	0.601
Firm Size	✓	✓	✓	✓	✓
Industry FE	✓	✓	-	-	-
Year FE	-	✓	-	-	-
Industry*Year FE	-	-	✓	✓	✓

Note: The dependent variable is the log of total assets at the segment-year level. *multi-segment* is a dummy with a value of one for multi-segment firms and zero for single-segment firms, *marginal costs* denotes relative marginal costs in each segment, adjusted for measurement error and purged of common factors. *low (high) friction* refer to firms below (above) the median in terms of number of boards the average CEO sits on (column 4), or below (above) the median in terms of the years the average CEO sits on the board (column 5). Standard errors are clustered at the firm level. *** p<0.01, ** p<0.05, * p<0.1

Column (1) in Table 1 shows that single-segment firms with higher marginal costs are allocated less capital ($\gamma_2 < 0$) than single-segment firms with lower marginal costs. The positive and significant interaction term indicates that, conditional on the same core marginal costs, multi-segment firms allocate more capital to segments than single-segment firms ($\gamma_3 > 0$). Adding year fixed effects to account for time trends in column (2), or industry*year fixed effects that absorb any unobservable trends that affect all firms within the same industry in column (3), does not change this pattern. In terms of magnitude, comparing segments at the 90^{th} to those at the 10^{th} percentile in terms of marginal costs, multi-segment firms allocate around 19% more assets to the less efficient segment, relative to single-segment firms with similar marginal costs. Columns (1)–(3)

thus provide support for the relation stated in Prediction 1.1.

In columns (4)–(5) we investigate Prediction 1.2 and the role of informational frictions. Our main measure of informational frictions is the average number of of boards the firms' CEOs sit on. The intuition is that CEOs serving on boards of several companies are less familiar with individual segments of these companies. Column (4) shows that capital misallocation is worse among multi-segment firms subject to more severe frictions. When we interact our main variables with a dummy *high friction* that takes on a value of one for firms whose CEOs sit on an above-median number of boards, the coefficient on the triple interaction effect is positive and highly significant. Column (5) introduces an alternative measure of informational frictions, based on a split by the time that the average CEO spent on the board of a firm. Firms in which the average CEOs has spent comparatively little time on the board are classified as *high friction* firms; firms in which the average CEO served a relatively long time on the board are classified as *low friction*. Similar to column (4), multi-segment firms with more severe informational frictions allocate more capital to segments relative to single-segment firms with comparable marginal costs – as seen from the positive triple interaction effect, significant at the 1% level. These findings suggest that the problem of allocating too much capital to the best segments (relative to single-segment firms) is stronger for multi-segment firms in which informational frictions are more severe, supporting Prediction 1.2.

5.3 The China shock and within firm capital allocation

We now investigate how an exogenous shock to competition − rising imports from China − affects within-firm capital allocation and the conglomerate discount. In the model, tougher import competition (modeled as a decline in trade costs) reduces the cost level at which divisions can survive in the market. As managers' scope for over-reporting is constrained by c_D, competition reduces over-reporting. The incentive to over-report declines most for managers in segments with the lowest relative marginal costs, which follows directly from Proposition 1. The fall in marginal costs in the best segments, relative to worse segments, implies that there is an increase in dispersion in marginal costs across segments. Equation 10 shows that such a wider dispersion of marginal costs within firms increases multi-segment firm's Q and hence reduces the conglomerate discount, which results in the following prediction:

Prediction 2.1 *An increase in import competition reduces the conglomerate discount.*

Competition also affects the headquarters' allocation of capital across segments. Equation 12b shows that the overall change in asset allocation depends on the relative strength of two components: the *demand effect* and the *over-reporting effect*:

$$\underbrace{\left.\frac{\partial \ln\left(asset_{fst}\right)}{\partial \ln\left(c_{Dt}\right)}\right|_{\ln\left(rmc_{fst}\right)}}_{\text{DEMAND EFFECT}} = 2 \tag{14}$$

$$\underbrace{\left.\frac{\partial \ln\left(asset_{fst}\right)}{\partial \ln\left(rmc_{fst}\right)}\right|_{\ln\left(c_{D}\right)}}_{\text{OVER-REPORTING EFFECT}} = \frac{1-2\ rmc_{fst}}{1-rmc_{fst}} \tag{15}$$

The *demand effect* captures that lower trade costs increase segment sales and thereby directly segment assets, conditional on marginal costs. The *over-reporting effect* captures the indirect effect of lower c_{Dt} through lower marginal costs on segment assets. In response to a trade shock over-reporting declines. If the elasticity of μ_{fst} with respect to a change in c_D is larger than one, segment marginal costs fall, and fewer assets are allocated to the respective segment. This effect will be particularly strong for the best segments, due to the hump-shaped relationship between assets and marginal costs. The overall effect of competition on capital allocation thus depends on which effect dominates,[21] which yields the following prediction:

Prediction 2.2 *An increase in import competition increases the allocation of capital to the best segments if the demand effect dominates the over-reporting effect.*

To test Prediction 2.1–Prediction 2.2 empirically, we follow Autor et al. (2013) and define $\Delta China_i$ as the change in import penetration for four-digit SIC industry i from 1999–2007. Industries with a stronger increase in Chinese imports are subject to tougher competition. Yet, simple OLS regression could suffer from omitted variable bias or reverse

[21]Note that the derivative of log assets with respect to log rmc (over-reporting effect) in Equation 15 is positive for $rmc \leq 0.5$ and surpasses -2 (the direct effect of c_D on log assets) only at around $rmc \geq 0.7$. This means that the over-reporting effect is expected to dominate the demand effect in the least efficient segments.

causality. For example, imports from China could rise the most in industries where domestic (US) multi-segment firms are of particularly low quality. To address these concerns and isolate the supply component in observed imports, we instrument actual US imports from China with imports from China to eight other advanced economies. We will also show that our results are robust to accounting for differential effects of potentially confounding factors, such as declining interest rates.

We first investigate Prediction 2.1, i.e. the effect of competition on the conglomerate discount and estimate the following firm-level regression from 1999–2007:

$$\begin{aligned}\Delta Q_{fit} = \delta_1 \; multi\ segment_f + \delta_2 \; \Delta Chinese\ imports_i \\ + \delta_3 \; multi\ segment_f \times \Delta Chinese\ imports_i + \zeta_f + \theta_i + \tau_t + \epsilon_{fit}.\end{aligned} \tag{16}$$

The dependent variable ΔQ denotes the yearly change in the Tobin's Q of firm f in industry i. The dummy *multi segment* takes on a value of one if firm f is a multi-segment firm, $\Delta Chinese\ imports_i$ denotes the (instrumented) change in Chinese imports from 1999–2007 in industry i. We control for firm size through dummies for quartiles of the firm employment distribution. To control for factors common to firms within the same industries, as well as common trends over time, we subsequently add industry and time fixed effects. All regressions cluster standard errors at the industry level, i.e. the level of the shock. The coefficient δ_1 indicates the change in conglomerate discount for the average multi-segment firm, absent any change in competition. Coefficient δ_3 indicates how Q changes for multi-segment firms in industries with a stronger increase in competition, relative to single-segment firms. Prediction 2.1 implies $\delta_3 > 0$: the conglomerate discount declines by more in industries with a stronger rise in Chinese imports.

Table 2, column (1) shows that the average Q grows faster for multi-segment firms, relative to single-segment firms (significant at the 10% level). In other words, the conglomerate discount declined over the sample period in industries with no change in imports from China. Coefficient δ_3 is positive and significant at the 1% level, indicating that the discount declines faster in industries with a stronger rise in competition. Adding industry fixed effects in columns (2) and year fixed effects in column (3) does not materially affect the estimated coefficients. Note that industry fixed effects absorb the coefficient on $\Delta Chinese\ imports$. Results thus suggest that an increase in competition reduces the conglomerate discount. In terms of magnitude, in column (3) a one standard deviation increase in $\Delta Chinese\ imports$ leads to a 4.5% increase in Q among multi-segment firms,

Table 2: **Import competition and the conglomerate discount**

VARIABLES	(1) Δ Q	(2) Δ Q	(3) Δ Q
multi-segment	0.041*	0.031	0.031
	(0.023)	(0.024)	(0.021)
Δ Chinese imports	-0.070		
	(0.164)		
multi-segment × Δ Chinese imports	0.342***	0.273**	0.242**
	(0.129)	(0.112)	(0.107)
Observations	2,018	2,018	2,018
Firm Size	✓	✓	✓
Industry FE	-	✓	✓
Year FE	-	-	✓

Note: The dependent variable is the yearly change in firms' Tobin's Q. *multi-segment* is a dummy with a value of one for multi-segment firms and zero for single-segment firms, Δ *Chinese imports* denotes the change in Chinese import penetration at the industry level from 1999 to 2007, instrumented with Chinese imports in 8 other advanced economies. All regression cluster standard errors at the firm level and are weighted by firm sales. The mean (sd) of the dependent variable is -0.02 (0.33). Δ *Chinese imports* has a mean (sd) of 0.11 (0.19). *** p<0.01, ** p<0.05, * p<0.1

relative to single-segment firms. In other words, the conglomerate discount narrows in response to competition. Note that coefficients in Table 2 show the differential effect of competition on Q of multi-segment firms in a given year. As the China shock spans the period from 1999 to 2007, column (3) implies a cumulative effect of competition on Q of around one-third over the eight year period.

To investigate Prediction 2.2, i.e., the effect of competition on the allocation of assets among multi-segment firms, we estimate the following segment-level regression for the sample of multi-segment firms only:

$$\begin{aligned}\Delta assets_{fsit} =& \xi_1 \ \textit{efficient segment}_{fs} + \xi_2 \ \Delta Chinese\ imports_i \\ &+ \xi_3 \ \textit{efficient segment}_{fs} \times \Delta Chinese\ imports_i + \zeta_f + \tau_t + \epsilon_{fsit},\end{aligned} \tag{17}$$

where $\Delta assets$ is the yearly change in the log of total assets of segment s belonging to firm f in segment industry i. $\Delta Chinese\ imports$ is the 1999–2007 (instrumented) change in import penetration of segment industry i, *efficient segment* is a dummy that takes on a value of one if a segment has the lowest marginal costs among all segments of a firm. Each regression controls for firm size through dummies for quartiles of the firm employment distribution (ζ_f) and for common trends through time fixed effects (τ_t).

The main coefficient of interest is ξ_3: the overall effect of competition on segment assets depends on the relative strength of the demand vs. over-reporting effect (Prediction 2.2). If $\xi_3 > 0$, this implies that an increase in import competition increases allocated capital to the best segments and that the demand effect dominates the over-reporting effect.

The underlying friction that gives rise to an inefficient allocation of capital is asymmetric information. We thus expect the disciplining effects of import competition on segment marginal costs and assets to be particularly strong within firms that are subject to stronger informational asymmetries. Similar to the firm level analysis, we classify firms into those with high and low informational frictions along the median number of boards the firms' CEOs sit on, as well as the median tenure of CEOs on the board.

Table 3 shows that rising import competition leads to an increase in allocated assets to efficient segments of multi-segment firms. Column (1) shows a an increase in assets allocated to better segments in response to tougher competition, indicated by the positive and significant interaction term. Adding industry fixed effects does not affect coefficients, see column (2). From Prediction 2.2 it follows that the demand effect dominates the over-reporting effect, as the coefficient on the interaction term (ξ_3) suggests is positive. How large is the effect of rising imports on allocated assets? In column (2), an increase in import competition by one percentage point increases assets allocated to the best segments by around 17.9%.

The competition-induced increase in asset allocation is particularly strong among multi-segment firms with more severe frictions. Adding interaction effects with the measures of informational friction, i.e. number of boards and tenure, in columns (3) and (4) shows that the increase in assets is more pronounced in firms in which informational frictions are higher and the scope for a reduction in over-reporting larger.

Taken together, the results in Table 3 suggest that rising competition leads to an increase in the efficiency of the internal capital market by increasing assets allocated to the best segments. The disciplining effect of competition is stronger for firms with more severe informational frictions, relative to firms with lower frictions.

Table 3: **Import competition and within-firm capital allocation**

VARIABLES	(1) Δ assets	(2) Δ assets	(3) nr boards Δ assets	(4) time Δ assets
efficient segment	-0.017	-0.016	-0.042***	0.022*
	(0.011)	(0.011)	(0.016)	(0.012)
Δ Chinese imports	-0.035	-0.058	0.031	-0.026
	(0.054)	(0.054)	(0.072)	(0.077)
efficient segment × Δ Chinese imports	0.182**	0.179**	-0.142	-0.246**
	(0.074)	(0.074)	(0.104)	(0.103)
high friction			-0.055**	-0.042**
			(0.023)	(0.017)
efficient segment × high friction			0.050**	-0.141***
			(0.021)	(0.022)
Δ Chinese imports × high friction			-0.073	0.087
			(0.134)	(0.102)
efficient segment × Δ Chinese imports × high friction			0.723***	1.156***
			(0.148)	(0.150)
Observations	1,909	1,909	1,909	1,909
Firm Size	✓	✓	✓	✓
Industry FE	-	✓	✓	✓
Year FE	✓	✓	✓	✓

Note: The dependent variable is the change log assets ($\Delta assets$). *efficient segment* is a dummy with value one for the most-efficient segments within a firm in terms of average return on assets. Δ *Chinese imports* denotes the change in Chinese import penetration at the industry level from 1999 to 2007, instrumented with Chinese imports in 8 other advanced economies. *low (high) friction* refer to firms below (above) the median in terms of number of boards the average CEO sits on, or below (above) the median in terms of the years the average CEO sits on the board. All regression are weighted by segment sales. The mean (sd) of the dependent variable is 0.06 (0.18) for $\Delta assets$. Δ *Chinese imports* has a mean (sd) of 0.07 (0.15). *** p<0.01, ** p<0.05, * p<0.1

Robustness In the online appendix, we further control for the cost of external financing and the average wage in each industry. In our model, a lower interest rate could lead to the financing of a worse pool of segments and conglomerate firms, as firms finance segments with higher levels of over-reporting. Moreover, in the model the average outside wage corresponds to the guaranteed compensation of divisional managers determined in the outside market. Holding sectoral wages or the outside cost of financing constant does not change our results.

6 Conclusion

This paper embeds an internal capital market into a model of multi-product firms with monopolistic competition. Internal capital markets inefficiently allocate capital due to the empire building motive of divisional managers in firms. Managers compete for internal capital and overstate their divisions' true costs to receive more funds. The best divisions in each firm end up getting "too much" capital, and stronger information asymmetries between firm owners and managers exacerbate such mis-allocation.

By embedding an internal capital market in a theory of multi-product firms, our model provides insights on how international trade affects conglomerates. In particular, it shows how fiercer competition can discipline managers, improve the efficiency of firms' internal capital market, and reduce the conglomerate discount. We find support for the models' predictions in a sample of publicly listed US firms.

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Appendix: For Online Publication

7 Appendix A

This appendix outlines definitions and proofs stated in Section 3.

7.1 Private benefit

A manager at a firm with core marginal cost c endowed with true customization cost x_i and reporting a customization cost z_i runs production according with the technology (2) given the allocation of labor and capital $l(z_i c)$ and $k(z_i c)$, from (3d) and (3e) respectively, and a total factor productivity $(x_i c)^{-1}$ instead of $(z_i c)^{-1}$. This yields output equal to $\frac{z_i}{x_i} y(z_i c)$. The division sells $q(z_i c) = y(z_i c)$ units in the market. Thus, defining the private benefit as the excess of output capacity $\frac{z_i}{x_i} y(z_i c) - q(z_i c)$ yields:

$$b\left(\mu; x_i, c, c_D\right) = \frac{L}{2\gamma}\left(\mu - 1\right)\left(c_D - \mu x_i c\right) \ .$$

The private benefit is null at $\mu = 1$, it is positive for $\mu < \frac{c_D}{x_i c}$ and it has a maximum at $\bar{\mu} \equiv \frac{1}{2}\left(1 + \frac{c_D}{x_i c}\right)$ as shown by the first and second order derivatives:

$$\frac{\partial b}{\partial \mu} = \frac{L}{2\gamma}\left(c_D + x_i c - 2\mu x_i c\right) \qquad \frac{\partial^2 b}{\partial^2 \mu} = -\frac{L}{\gamma} x_i c < 0 \ .$$

Indeed, $\frac{\partial b}{\partial \mu} > 0$ for every $\mu < \bar{\mu}$. This is without loss of generality, since it would never be optimal a factor of mis-reporting that exceeds $\bar{\mu}$, nor that is equal to $\bar{\mu}$ since the manager maximises the expected payoff and the probability of being financed is decreasing in the factor of mis-reporting.

The comparative statics of the equilibrium are robust to a more general definition of private benefit:

Definition. *Manager's private benefit is additional welfare represented by any continuous and twice differentiable function $b\left(\mu; x_i, c, c_D\right)$ with the same qualitative comparative statics as the excess of output capacity:*

$$b(1) = 0 \qquad \frac{\partial b}{\partial \mu} > 0 \qquad \frac{\partial^2 b}{\partial^2 \mu} < 0 \ , \tag{18}$$
$$\frac{\partial^2 \ln(b)}{\partial \mu \partial x_i} < 0 \qquad \frac{\partial^2 \ln(b)}{\partial \mu \partial c} < 0 \qquad \frac{\partial^2 \ln(b)}{\partial \mu \partial c_D} > 0 \ ,$$

where $\mu = z_i/x_i$ is the factor of mis-reporting.

While the first line of the comparative statics are trivial to asses, the second line refers to the log transformation of the excess of output capacity:

$$\frac{\partial \ln(b)}{\partial \mu} = \frac{1}{\mu - 1} - \frac{1}{\frac{c_D}{x_i c} - \mu}$$

which is decreasing in $x_i c/c_D$. As it can be verified in the following proof of Proposition 1, the general definition of private benefit (18) provides the sufficient conditions for a determination of the equilibrium.■

7.2 Proof of the Lemma

This paragraph proves the Lemma and its result (7). Let $J(z|m)$ be the probability that $z_i \leq z$ among managers that compete for m positions. The probability of m independent events $z_i \leq z$ conditional on $z_i \leq \bar{z}(c, c_D)$ is $\left(\frac{J(z|m)}{J(\bar{z}(c,c_D)|m)}\right)^m$. The complement at 1 is the probability that a manager in the pool of those competing for m positions at a firm with core marginal cost c and offering z is financed:

$$\psi(\mu; x_i, c, m, c_D) = 1 - \left(\frac{J(\mu x_i|m)}{J(\bar{z}(c, c_D)|m)}\right)^m \quad (19)$$

Taking the first order derivative yields:

$$\frac{\partial \psi}{\partial \mu} = -m \left(\frac{J(\mu x_i|m)}{J(\bar{z}(c, c_D)|m)}\right)^m \frac{dJ(\mu x_i|m)}{dz} \frac{x_i}{J(\mu x_i|m)} < 0 \,, \quad (20)$$

where the number of non-core divisions is $m \geq 1$.

If $J(z|m)$ is convex on the support $z \in [0, z_{max}]$ then $\frac{\partial^2 \psi}{\partial^2 \mu} \leq 0$ and ψ is decreasing in μ and in x_i, decreasing in c and increasing in c_D, as implied by $\frac{\partial \bar{z}(c,c_D)}{\partial c} < 0$ and $\frac{\partial \bar{z}(c,c_D)}{\partial c_D} > 0$.[22] The comparative statics of the probability of being financed with respect to μ, x_i, c, and c_D are proved.

For the dependence with respect to the number of non-core divisions the assumption of no economies of scope is necessary.[23] For a given managerial compensation scheme,

[22]A necessary condition for the comparative statics derived in this lemma is $(m-1)\left(\frac{dJ(z|m)}{dz}\right)^2 + J(z|m)\frac{d^2 J(z|m)}{dz^2} > 0$ which implies $\frac{\partial^2 \psi}{\partial^2 \mu} \leq 0$. Convexity of the function $J(z|m)$ clearly satisfies the requirement as a sufficient condition, but the results apply to a broader class of functions.

[23]Technically, the outcome of the tournament process between managers is described as a Nash Bayesian equilibrium in reported customisation costs announced by managers. An explicit and full characterization of this type of equilibrium is intractable. The assumption of no economies of scope

the expected value to the firm from a non-core division in a MSF with core marginal cost c opening m divisions given a market cutoff c_D is:

$$\bar{v}_{nc}(m, c, c_D) = \int_1^{\bar{z}(c,c_D)} [(1-\eta)\pi(zc) - f_M] \frac{mJ(z|m)^{m-1}}{J(\bar{z}(c, c_D)|m)^m} dJ(z|m) \ . \tag{21}$$

A necessary and sufficient condition to rule out economies of scope is that $m\bar{v}_{nc}(m, c, c_D)$ does not depend on the number of non-core divisions m, for every $m = 1, 2, ...$.

In order to simplify the exposition while determining the implications of that assumption for the conditional distribution of customisation costs, call $\Gamma(a, m) \equiv m \int_1^a [(1-\eta)\pi(zc) - f_M] \frac{mJ(z|m)^{m-1}}{J(\bar{z}|m)^m} dJ(z|m)$. The condition such that $m\bar{v}_{nc}(m, c, c_D)$ does not depend on m can be expressed as $\frac{\partial\Gamma(a,m)}{\partial m} = 0 \ \forall a$ which implies $\frac{\partial^2\Gamma(a,m)}{\partial m \partial a} = 0 \ \forall a$. Computing the first order derivative

$$\frac{\partial\Gamma(a, m)}{\partial a} = m[(1-\eta)\pi(ac) - f_M] \frac{mJ(a|m)^{m-1}}{J(\bar{z}|m)^m} dJ(a|m) \ ,$$

shows that the second order cross derivative is null if and only if

$$\frac{\partial^2\Gamma(a, m)}{\partial m \partial a} = 0 \iff \frac{\partial}{\partial m}\left(m \frac{mJ(a|m)^{m-1}}{J(\bar{z}|m)^m} dJ(a|m)\right) = 0 \ \forall a.$$

This is true if and only if the density of m independent draws from the conditional distribution $J(z|m)$ truncated at $\bar{z}$ is equal to a marginal density truncated at $\bar{z}$ which does not depend on the number of non-core divisions:[24]

$$m \frac{mJ(a|m)^{m-1}}{J(\bar{z}|m)^m} dJ(a|m) = \frac{dF(a)}{F(\bar{z})} \ ,$$

where $F : [1, \infty) \to [0, 1]$ is a continuous and increasing function; moreover, notice that $J(z|m)$ convex in $z \geq 1$ implies that also $F(z)$ is convex in $z \geq 1$.

Integrating both sides of the differential equation over the support $a \leq 1$ and over the support $a \leq \bar{a}$ for any arbitrary $\bar{a} \in [1, \bar{z}]$ yields respectively

$$\begin{aligned} \left(\frac{J(1|m)}{J(\bar{z}|m)}\right)^m &= \frac{1}{m}\frac{F(1)}{F(\bar{z})} \\ \left(\frac{J(\bar{a}|m)^m - J(1|m)^m}{J(\bar{z}|m)^m}\right) &= \frac{1}{m}\left(\frac{F(\bar{a}) - F(1)}{F(\bar{z})}\right) \ . \end{aligned}$$

facilitates a characterization of managers' misreporting behavior and the analysis of how it is affected with respect to competition and characteristics of the firm.

[24]To see this it is sufficient to substitute for $m = 1$ in the left hand side and then recall that the result must be the same for every arbitrary number m.

The system of the two conditions yields:

$$\frac{J(\bar{a}|m)}{J(\bar{z}|m)} = \left(\frac{F(\bar{a})}{mF(\bar{z})}\right)^{\frac{1}{m}} \quad \forall \bar{a} \in [1, \bar{z}].$$

Now, applying this result to our problem shows that there are no economies of scope for every arbitrary firm and market conditions (i.e. for every c and c_D) if and only if:

$$\frac{J\left(z|m\right)}{J(\bar{z}(c,c_D)|m)} = \left(\frac{F(z)}{mF(\bar{z}(c,c_D))}\right)^{\frac{1}{m}} \quad , \ \forall \ z < \bar{z}(c,c_D) \ . \tag{22}$$

Equation (22) shows that the assumption of $J(z|m)$ being convex on $z \in [1, z_{max}]$ is equivalent to the assumption about $F(z)^{\frac{1}{m}}$ in the statement of the Lemma. Therefore, the less restrictive necessary condition that we have discussed earlier in this paragraph applies to the function $F(z)^{\frac{1}{m}}$ as well.

Substituting for (22) in (19) yields:

$$\begin{aligned} \psi(\mu; x_i, c, m, c_D) &= 1 - \frac{F(\mu x_i)}{mF(\bar{z}(c,c_D))} \\ \frac{\partial \psi}{\partial \mu} &= -\frac{x_i}{mF(\bar{z}(c,c_D))}\frac{dF(\mu x_i)}{dz} < 0 \ . \end{aligned}$$

Thus, the probability of being financed is increasing in non-core divisions m. ■

Before proceeding with the proof of Proposition 1, we show that the characterization of the probability of being financed implies an expected profit from non-core divisions that is equal to the average profit of a non-core segment. Rearranging (22) yields:

$$\frac{J\left(z|m\right)}{J(\bar{z}(c,c_D)|m)} = \left(\frac{F(z)}{mF(\bar{z}(c,c_D))}\right)^{\frac{1}{m}} \qquad \frac{dJ(z|m)}{J\left(z|m\right)} = \frac{dF(z)}{mF(z)} \quad , \ \forall \ z < \bar{z}(c,c_D) \ .$$

Substituting in (21) yields the expected value to the firm from m non-core divisions:

$$m\bar{v}_{nc}(c,m,c_D) = \int_1^{\bar{z}(c,c_D)} \frac{[(1-\eta)\pi(zc) - f_M]dF(z)}{F(\bar{z}(c,c_D))} \ , \tag{23}$$

which does not depend on the actual number of non-core divisions m.

7.3 Proof of Proposition 1

As announced in the main body of the paper, we conduct the proof allowing for an arbitrary share of profit $\eta \in [0,1]$. This is redundant in equilibrium, since MSFs would optimally set $\eta = 0$, but it is instructive to show how pecuniary incentives affect managers' behavior. The proof is organized in four lemmas and then the main statement of the proposition.

Individual welfare (**??**) is linear in income, thus, the pecuniary incentive of a profit share adds to the private benefit. This modifies the manager's expected payoff, which is now given by the private benefit plus the share of profit. The manager's problem (8) that is now replaced by:

$$\mu_i^\star = \arg\max_{\mu\geq 1} \ \psi\left(\mu; x_i, c, c_D, m\right)\left[b\left(\mu; x_i, c, c_D\right) + \eta\pi(\mu x_i c)\right] \ , \tag{24}$$

The manager's problem (24) has a unique solution, which is characterized by the following generalization of Proposition 1:

Generalization of Proposition 1. *Better managers, i.e. those characterized by* $1 \leq x_i < \frac{c_D}{2c}$*, over-report the customization cost* $\mu_i^\star\left(\eta; x_i, c, m, c_D\right) > 1$*, and the the comparative statics describing their optimal decision are such that:*
(1.1) $\mu_i^\star$ *is decreasing in the true customization cost of the division* x_i
(1.2) $\mu_i^\star$ *is decreasing in the core competence cost of the firm* c
(1.3) $\mu_i^\star$ *is increasing in the number of non-core divisions* m
(1.4) $\mu_i^\star$ *is increasing in the market cutoff cost* c_D
(1.5) $\mu_i^\star$ *is decreasing in the share of profit paid to the manager* η

instead, worse managers, i.e. those characterized by $\frac{c_D}{2c} \leq x_i < \frac{c_M(c_D)}{c}$*, report the effective customization cost.*

Clearly, the Generalization of Proposition 1 coincides with Proposition 1 but for the comparative statics (1.5) on the profit share.

The characterization of a manager's private benefit (18) is sufficient to prove the following result:

Lemma 1.1. *The semi-elasticity* $\xi_b(\mu) \equiv \frac{\partial b/\partial\mu}{b}$ *goes to infinity at* $\mu = 1$*, is decreasing in* μ*,* x_i*,* c *and is increasing in* c_D*.*

Proof. The semi-elasticity $\frac{\partial b/\partial\mu}{b}$ is decreasing in μ, since $\frac{\partial^2 b}{\partial^2\mu} < 0$ at the numerator and

$\frac{\partial b}{\partial \mu} > 0$ at the denominator. For $\mu = 1$ the semi-elasticity goes to infinity, as $b(1) = 0$ while $\frac{\partial b(1)}{\partial \mu}$ is positive and finite. To determine the comparative statics with respect to x_i, c and c_D we must look at the functional form of the semi-elasticity as implied by the excess of output capacity:

$$\xi_b(\mu) \equiv \frac{d\ln(b(\mu))}{d\mu} = \frac{\partial b/\partial \mu}{b} = \frac{1 + \frac{x_i c}{c_D} - 2\mu\frac{x_i c}{c_D}}{\mu\left(1 + \frac{x_i c}{c_D}\right) - \mu^2\frac{x_i c}{c_D} - 1} .$$

It is sufficient taking the derivative with respect to $x_i c/c_D$ to conclude that the semi-elasticity ξ_b is decreasing in x_i and c while it is increasing in c_D. ■

The characterization of the probability to be financed (7) is sufficient to prove the following result:

Lemma 1.2. *The semi-elasticity $\xi_\psi(\mu) \equiv -\frac{\partial\psi/\partial\mu}{\psi}$ takes a positive but finite value at $\mu = 1$, it is increasing in μ, x_i and c while it is decreasing in c_D and m.*

Proof. The semi-elasticity $-\frac{\partial\psi/\partial\mu}{\psi}$ is increasing in μ, as implied by $-\frac{\partial^2\psi}{\partial^2\mu} < 0$ at the numerator and $\frac{\partial\psi}{\partial\mu} < 0$ at the denominator.

The semi-elasticity is given by:

$$\xi_\psi(\mu) \equiv -\frac{\partial\psi/\partial\mu}{\psi(\mu; x_i, c, m, c_D)} = \frac{x_i}{mF(\bar{z}(c, c_D)) - F(\mu x_i)}\frac{dF(\mu x_i)}{dz} ,$$

which is increasing in μ and in x_i for every exogenous convex c.d.f. $F(z)$, increasing in c and decreasing in c_D, as implied by $\frac{\partial\bar{z}(c,c_D)}{\partial c} < 0$ and $\frac{\partial\bar{z}(c,c_D)}{\partial c_D} > 0$, and it is decreasing in the number of non-core divisions m. Since $x_i > 1$ and $F(x_i) < mF(\bar{z}(c, c_D))$ then both $\psi(1)$ and $-\frac{\partial\psi(1)}{\partial\mu}$ are positive and finite values. Thus, the semi-elasticity $-\frac{\partial\psi/\partial\mu}{\psi}$ takes a positive but finite value for $\mu = 1$ for every number $m = 1, 2, ...$ of non-core divisions. ■

Lemma 1.3. *The semi-elasticity of the manager's payoff $\xi_{pay}(\mu)$ goes to infinity at $\mu = 1$, is decreasing in μ, x_i, c, η and is increasing in c_D.*

Proof. Given the equilibrium profit (3f), the semi-elasticity of profit with respect to the factor of mis-reporting is

$$\xi_\pi(\mu) \equiv \frac{\partial\pi/\partial\mu}{\pi} = -\frac{2\frac{x_i c}{c_D}}{1 - \mu\frac{x_i c}{c_D}} < 0 .$$

Thus, $\xi_\pi(\mu)$ is negative, decreasing in $\frac{x_i c}{c_D}$ which is the same comparative static as for $\xi_b(\mu)$. The semi-elasticity of manager's payoff is a convex combination of the semi-elasticity of the private benefit and of profit:

$$\xi_{pay}(\mu) \equiv \xi_b(\mu)(1-\omega(\mu,\eta)) + \omega(\mu,\eta)$$

where the weight of profit in the manager's payoff

$$\omega(\mu,\eta) \equiv \frac{\eta\pi}{b+\eta\pi} = \frac{\eta(c_D-\mu x_i c)}{2(\mu-1)+\eta(c_D-\mu x_i c)} \ ,$$

is increasing in η and c_D, decreasing in μ, x_i and c, which are the same signs than the comparative statics for $\xi_b(\mu)$.

Since $\xi_\pi(\mu) < 0$ the semi-elasticity of manager's payoff with respect to the factor of mis-reporting is lower or equal to the semi-elasticity of manager's private benefit

$$\xi_{pay}(\mu) \equiv \xi_b(\mu)(1-\omega(\mu,\eta)) + \omega(\mu,\eta)\xi_\pi(\mu) \leq \xi_b(\mu) \ .$$

Since $\xi_{pay}(\mu)$ corresponds to $\xi_b(\mu)$ if and only if $\eta = 0$, then, everything else being constant, $\xi_\pi(\mu) \leq \xi_b(\mu)$ and $\xi_{pay}(\mu)$ is a decreasing function of η. The comparative statics with respect to x_i, c and c_D are affected by the introduction of a profit share. However, changes in $\xi_\pi(\mu)$ are of the same sign as changes in $\xi_b(\mu)$. And the same is true for $\xi_{pay}(\mu)$ being a convex combination of two functions both decreasing in x_i, c, and increasing in c_D.

We now turn to study $\xi_{pay}(\mu)$. Consider $\mu = 1$, then $\xi_b(1) \to \infty$ while $\xi_\pi(1)$ is negative, but finite. This implies $\xi_{pay}(1) \to \infty$. For $\mu > 1$ but such that it satisfies profitability condition $\mu x_i c < c_D$, the positive elasticity $\xi_b(\mu)$ falls and $\xi_\pi(\mu)$ becomes even more negative, such that the convex combination $\xi_{pay}(\mu)$ has to be decreasing for every $\mu \geq 1$. Although mathematically possible, there is no need to consider the case in which $\xi_{pay}(\mu) < 0$ because it is not in the interest of the manager to over-report so much to overcome the arg max of the payoff function, corresponding to a semi-elasticity equal to zero. ■

Lemma 1.4. *Only managers with a sufficiently good know-how such that*

$$1 \leq x_i < \frac{c_D}{2c} \tag{25}$$

are capable to extract a private benefit in a firm with core marginal cost c when the market cutoff is c_D.

Proof. The equilibrium allocation of capital (3e) shows that $k(z_i c) > k(x_i c)$ if and only if:

$$(c_D - z_i c)z_i c < (c_D - x_i c)x_i c \iff \frac{(\mu+1)(\mu-1)}{\mu-1} < \frac{c_D}{x_i c} \iff \mu < \frac{c_D}{x_i c} - 1 \ .$$

Since $b(\mu) \geq 0 \iff \mu \geq 1$, then a necessary condition for both $b(\mu) \geq 0$ and $k(z_i c) > k(x_i c)$ is

$$\frac{x_i c}{c_D} < \frac{1}{2} \ .$$

We call *better managers* those capable of an effective customization cost x_i that satisfies (25). We distinguish them from *worse managers*, endowed with a higher but profitable customization cost $\frac{c_D}{2c} \leq x_i < \frac{c_M(c_D)}{c}$, who are still suitable to run a division at the firm, but they cannot both engage in cost over-reporting $\mu_i > 1$ and managing more capital $k(z_i c) > k(x_i c)$. ■

Proof of Proposition 1. The first order condition of the manager's problem (8) is necessary and sufficient for an interior solution:

$$\xi_{pay}(\mu) - \xi_{\psi}(\mu) = 0 \ ,$$

since $\xi_{pay}(\mu)$ is decreasing in μ by Lemma 1.1 and 1.3, while $\xi_{\psi}(\mu)$ is increasing in μ, by Lemma 1.2. An interior solution of the first order condition exists, it is unique and belongs to the segment $\mu > 1$, as it satisfies the intersection $\xi_{pay}(\mu) = \xi_{\psi}(\mu)$, where we have already established that $\xi_{pay}(1) > \xi_{\psi}(1)$.[25]

Lemma 1.4 has shown that only better managers gain a private benefit when they over-report, i.e. $\mu > 1$. The comparative statics of the solution $\mu_i^*(\eta, x_i, c, m, c_D)$ are unambiguously understood from the comparative statics of the two semi-elasticities $\xi_{pay}(\mu)$ and $\xi_{\psi}(\mu)$, discussed in the previous lemmas.

We have established that $\xi_{pay}(\mu)$ is a decreasing function of η for every $\mu \geq 1$, whereas $\xi_{psi}(\mu)$ does not depend on η. It follows the unique interior solution to the manager's problem (24) describes the optimal factor of over-reporting $\mu > 1$ as a decreasing function of the profit share η. The remaining comparative statics are discussed in the graphical analysis of figures (2a) and (2b).

Worse managers do not gain a strictly positive benefit. By over-reporting they would reduce both the probability of being financed and the profit of the division. Moreover, they do not under-report because that would lead to miss the targets, hence to a prohibitive punishment. Thus, for worse managers the only feasible solution is $\mu = 1$. ■

[25]The two curves need not be characterized by the concavity and convexity as shown in the Figure 2, since they would depend on the specific choice of private benefit function $b(\mu)$ and c.d.f. $F(z)$. However, for a private benefit function $b(\mu)$ and c.d.f. $F(z)$, the increasing and decreasing patterns are as drawn, and the marginal gain in expected payoff is higher than the marginal loss in the probability to be financed evaluated at $\mu = 1$.

8 Appendix B – further empirical evidence

Table 4 provides summary statistics for our main variables at the firm and segment level.

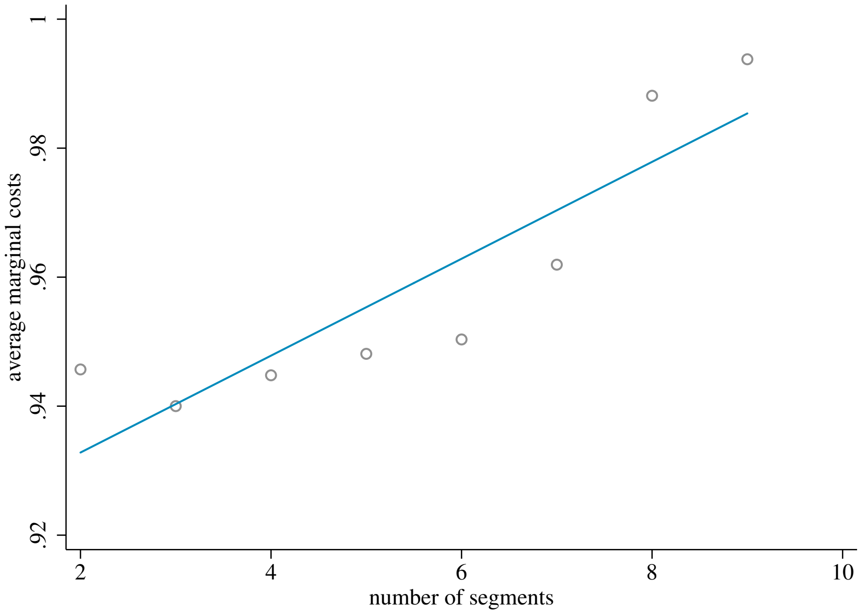
Table 4: **Descriptive statistics**

	mean	sd	min	max	count
Panel (a): Firm					
Q	1.65	0.69	0.59	6.87	3290
Panel (b): Segment					
multi-segment	0.83	0.38	0.00	1.00	16982
log(assets)	5.58	2.15	-5.90	13.26	16982
return on assets	0.12	0.37	-3.67	2.47	16982
marginal costs	-1.96	0.42	-4.22	-1.39	16982
Δ assets	0.56	0.98	-2.57	3.44	1909
Δ MC	0.00	0.34	-1.15	3.48	1909

Note: This table shows descriptive statistics for main variables. Sample size varies due to different levels of analysis.

Figure 3 shows that average marginal costs are higher among firms with more non-core segments. Table 5 controls for changes in the cost of external financing Our model and empirical results show how tougher import competition – measured through rising imports from China – reduce capital mis-allocation within firms. Yet, a potentially confounding factor could be declining interest rates, which have been linked to capital mis-allocation as well. For example, Gopinath et al. (2017) argue that lower interest rates in South Europe after the introduction of the Euro have increased mis-allocation of capital between firms. In our model, a lower interest rate could lead to the financing of a worse pool of segments and conglomerate firms, as firms finance segments with higher levels of over-reporting (result available on request). To account for this possibly confounding factor, in Table 5 we estimate Equation 17, but directly control for interest rates and their differential effect on assets and marginal costs of efficient segments. Specifically, we use Compustat data on the average interest expense over total debt for each 4-digit industry to proxy the evolution of financing costs among firms in each industry. Results show that accounting for the effect of changes in industry-level financing costs does not materially affect our results: tougher competition leads to a relative increase in assets among efficient segments. Investigating effects among firms with higher informational frictions, measured by the number of boards the average CEO sits on or the time the CEO spent in the company (columns 2–3) again shows that controlling for interest rates does not affect our main conclusion: effects of tougher competition on assets and marginal costs are generally stronger among firms with more severe informational frictions. Table 6 controls for the average wage in each industry, based on the NBER manufacturing data base, and shows that the coefficients of interest remain unaffected.

Figure 3: **Average marginal cost and number of segments**



Note: This figure provides a firm-level binned scatterplot of average marginal costs on the y-axis against the number of segments on the x-axis. Average marginal costs are higher among firms with more non-core segments.

Table 5: **Accounting for the cost of external financing**

VARIABLES	(1) Δ assets	(2) nr boards Δ assets	(3) time Δ assets
efficient segment	-0.084***	-0.102***	-0.057***
	(0.020)	(0.021)	(0.020)
Δ Chinese imports	-0.080	0.016	-0.038
	(0.054)	(0.072)	(0.077)
efficient segment × Δ Chinese imports	0.233***	-0.129	-0.215**
	(0.075)	(0.104)	(0.103)
high friction		-0.053**	-0.042**
		(0.023)	(0.017)
efficient segment × high friction		0.032	-0.129***
		(0.021)	(0.022)
Δ Chinese imports × high friction		-0.072	0.078
		(0.133)	(0.101)
efficient segment × Δ Chinese imports × high friction		0.806***	1.214***
		(0.149)	(0.150)
interest expense	2.078	2.050	0.432
	(1.306)	(1.297)	(1.284)
efficient segment × interest expense	4.519***	4.660***	5.129***
	(1.089)	(1.101)	(1.072)
Observations	1,909	1,909	1,909
Firm Size	✓	✓	✓
Industry FE	✓	✓	✓
Year FE	✓	✓	✓

Note: The dependent variable is the change in segment log assets ($\Delta assets$) or marginal costs (Δmc). *efficient segment* is a dummy with value one for the most-efficient segments within a firm in terms of average return on assets. Δ *Chinese imports* denotes the change in Chinese import penetration at the industry level from 1999 to 2007, instrumented with Chinese imports in 8 other advanced economies. All regression are weighted by segment sales. *low (high) friction* refer to firms below (above) the median in terms of number of boards the average CEO sits on, or below (above) the median in terms of the years the average CEO sits on the board. *** p<0.01, ** p<0.05, * p<0.1

Table 6: **Accounting for the average outside wage**

VARIABLES	(1) Δ assets	(2) nr boards Δ assets	(3) time Δ assets
efficient segment	-0.042	-0.072**	-0.024
	(0.032)	(0.034)	(0.032)
Δ Chinese imports	-0.060	0.039	-0.041
	(0.055)	(0.073)	(0.078)
efficient segment × Δ Chinese imports	0.168**	-0.166	-0.242**
	(0.075)	(0.106)	(0.104)
high friction		-0.054**	-0.038**
		(0.024)	(0.018)
efficient segment × high friction		0.047**	-0.151***
		(0.021)	(0.023)
Δ Chinese imports × high friction		-0.099	0.106
		(0.140)	(0.103)
efficient segment × Δ Chinese imports × high friction		0.744***	1.134***
		(0.149)	(0.151)
average wage	-0.072	-0.084	-0.100
	(0.066)	(0.067)	(0.065)
efficient segment × average wage	0.058	0.071	0.100
	(0.062)	(0.061)	(0.064)
Observations	1,876	1,876	1,876
Firm Size	✓	✓	✓
Industry FE	✓	✓	✓
Year FE	✓	✓	✓

Note: The dependent variable is the change in segment log assets ($\Delta assets$) or marginal costs (Δmc). *efficient segment* is a dummy with value one for the most-efficient segments within a firm in terms of average return on assets. Δ *Chinese imports* denotes the change in Chinese import penetration at the industry level from 1999 to 2007, instrumented with Chinese imports in 8 other advanced economies. *low (high) friction* refer to firms below (above) the median in terms of number of boards the average CEO sits on, or below (above) the median in terms of the years the average CEO sits on the board. All regression are weighted by segment sales. *** p<0.01, ** p<0.05, * p<0.1

9 Appendix C - online, not for publication

In this section of the appendix, we outline the derivations behind the results of the model that are discussed in the main body of the paper.

Consumer's problem

The consumer maximizes utility (1) subject to the budget constraint

$$\begin{aligned} U &= q_o^c + \alpha \int_0^V q_v^c dv - \frac{\beta}{2}\left(\int_0^V q_v^c dv\right)^2 - \frac{\gamma}{2}\int_0^V (q_v^c)^2 \, dv, \\ s.t. &: p_o q_o^c + \int_0^V p_v q_v^c dv \leq I^c \end{aligned}$$

where I^c is consumer's income. A strictly positive consumption of the outside good implies a constant marginal utility of income, that is equal to $1/p_o$. The outside good is the numeraire, therefore $p_o \equiv 1$.

Substituting in the first order condition for the consumption of differentiated good yields the inverse demand $p_v = \alpha - \gamma q_v^c - \beta Q^c$, thus, the linear uncompensated demand function

$$q_v^c = \frac{1}{\gamma}\left(\alpha - p_v - \beta Q^c\right) \ .$$

Integrating over the set of varieties V yields the household consumption of differentiated good $Q^c = (\alpha - \bar{p})\frac{V}{\gamma+\beta V}$, where $\bar{p} = \frac{1}{V}\int_0^V p_v dv$ is the average price across consumed varieties. Substituting back in the demand for a certain variety yields:

$$q_v^c = \frac{\alpha}{\gamma+\beta V} - \frac{1}{\gamma}p_v + \frac{\beta V}{\gamma+\beta V}\frac{1}{\gamma}\bar{p} \ .$$

By setting $q_v^c = 0$ yields the choke price that shuts down the demand of a certain variety, which is $p^{max} = \frac{1}{\gamma+\beta V}(\gamma\alpha + \beta V \bar{p})$. Thus, the demand for a certain variety can be written in terms of the choke price:

$$q_v^c = \frac{1}{\gamma}\left(p^{max} - p_v\right) \ .$$

Consumer's expenditure $\int_0^V p_v q_v^c dv$ is described my means of the choke price and by the first and second moments of the price distribution. The variance of prices is $\sigma_p^2 = \frac{1}{V}\int_0^V (p_v - \bar{p})^2 \, dv$, where $\bar{p} = \frac{1}{V}\int_0^V p_v dv$. Thus, the portion of consumer's income allocated to differentiated goods is $I^d = \bar{p}Q^c - \frac{V}{\gamma}\sigma_p^2$. Consumer's expenditure on the outside good is the residual $I_o^c = I^c - I_d^c > 0$.

Outside sector

The outside sector is competitive, thus the price p_o is equal to the marginal cost of production. The technology for producing an outside good employs labor and capital as perfect substitutes:

$$y_o = l + \theta k \ .$$

The implied cost function is $\min\{wy_o, \frac{p_k}{\theta} y_o\}$, where w is the price of one unit of labor and p_k is the rental price of one unit of capital. Both labor and capital are employed in the production of the outside good if and only if $p_k \equiv \theta w$. Under these circumstances, the marginal cost of one unit of outside good is equal to w. By perfect competition $w = p_o$, thus, the price of one unit of labor is equal to the price of the numeraire good $w = 1$ and the rental price of capital is $p_k = \theta$.

Firm's problem

The firm minimizes the cost subject to the technology (2)

$$\begin{aligned} C &= wl + \theta k \\ s.t. \quad &: \quad y = \frac{\varphi}{z_i c} \, l^{\lambda} k^{1-\lambda} \ . \end{aligned}$$

The equivalence between marginal rate of technical substitution $\frac{\lambda}{1-\lambda}\frac{k}{l}$ and relative price is $\frac{w}{\theta}$ yields the derived optimal factor demand:

$$\begin{aligned} l &= \frac{z_i c}{\varphi} \left(\frac{\theta}{w} \frac{\lambda}{1-\lambda} \right)^{1-\lambda} y \\ k &= \frac{z_i c}{\varphi} \left(\frac{\theta}{w} \frac{\lambda}{1-\lambda} \right)^{-\lambda} y \ . \end{aligned}$$

The cost function is equal to:

$$C = \left[w \left(\frac{\theta}{w} \frac{\lambda}{1-\lambda} \right)^{1-\lambda} + \theta \left(\frac{\theta}{w} \frac{\lambda}{1-\lambda} \right)^{-\lambda} \right] \frac{z_i c}{\varphi} y$$

thus, the normalization $\varphi \equiv w \left(\frac{\theta}{w} \frac{\lambda}{1-\lambda} \right)^{1-\lambda} + \theta \left(\frac{\theta}{w} \frac{\lambda}{1-\lambda} \right)^{-\lambda}$ is convenient to have the marginal cost of a product i in a firm with core competence cost c equal to $z_i c$. The normalization $\varphi_l = \left(\frac{\theta}{w} \frac{\lambda}{1-\lambda} \right)^{\lambda-1} \varphi$ and $\varphi_k = \left(\frac{\theta}{w} \frac{\lambda}{1-\lambda} \right)^{\lambda} \varphi$ is convenient to write the factor demands simply as $l = \frac{z_i c}{\varphi_l} y$ and $k = \frac{z_i c}{\varphi_k} y$.

A firm producing a certain variety which sells at a price p faces an aggregate demand that is L times the individual demand, thus $q = \frac{L}{\gamma} \left(p^{max} - p \right)$. Using the inverse demand

$p = p^{max} - \frac{\gamma}{L} q$ to obtain firm revenue yields a marginal revenue $p^{max} - \frac{2\gamma}{L} q$. The equivalence between marginal revenue and marginal cost implies the profit maximizing quantity, which, substituted in the inverse demand, yields the optimal price:

$$q = \frac{L}{2\gamma}(p^{max} - z_i c) \qquad p = \frac{1}{2}(p^{max} + z_i c) \ .$$

A variety that is produced as a core competence does not require any customization cost $z_0 = 1$. The demand of such a variety is zero if and only if $c = p^{max}$. Therefore, the core competence marginal cost above which firms do not make sales in their domestic market is $c_D \equiv p^{max}$. Substituting in the expression of optimal quantity and price leads to the equilibrium firm level performances (3c).

Aggregate variables

Core marginal cost $c \sim G(c)$ and customization cost $z \sim F(z)$ are two independent random variables. Their product $\nu = zc$ is a random variable whose realizations correspond to the marginal cost varieties $v \in [0, V]$. Let $H(\nu)$ be the exogenous cumulative density function of marginal cost, unconditional on a successful entry. The probability density function of marginal cost is:

$$h(\nu) = \int_0^\infty \frac{g(c) f(\nu/c)}{c} dc \ ,$$

where $g(c)$ and $f(z)$ are the probability density function of c and z respectively. Varieties that are successfully produced are a fraction $H(c_D)$ of the measure of potential entrants M_E, which implies:

$$M_E = \frac{V}{H(c_D)} \ .$$

The average marginal cost across varieties that make a successful entry is $\bar{\nu} = H(c_D)^{-1} \int_0^{c_D} \nu dH(\nu)$. This allows the average price to be computed $\bar{p} = \frac{1}{2}(c_D + \bar{v})$. Substituting for $c_D \equiv p^{max}$ in the expression $p^{max} = \frac{1}{\gamma + \beta V} (\gamma\alpha + \beta V \bar{p})$ determines the measure of varieties:

$$V = \frac{\gamma}{\beta} \frac{\alpha - c_D}{c_D - \bar{p}} \ .$$

The measure of incumbent firms M corresponds to the fraction of potential entrants with a core marginal cost $c \leq c_D$, that is:

$$M = G(c_D) M_E \ .$$

Firms with a core marginal cost $c \leq c_M(c_D)$ become a MSF, where the cutoff $c_M(c_D)$ is defined in (5). The average number of varieties per incumbent firm Ω and the average number of non-core segments per MSF are given by:

$$\Omega = \frac{V}{M} \qquad\qquad \bar{m} = \frac{V-M}{G(c_M(c_D))M} \ .$$

There are L many consumers. In equilibrium, a measure $V-M$ consists of divisional managers and $L-(V-M)$ are workers. Some workers are employed in the differentiated sector to foster firm entry $L_E = f_E M_E$, others in production $L_P = \bar{\ell}V$, where $\bar{\ell} = H(c_D)^{-1}\int_0^{c_D} \ell(\nu)dH(\nu)$ and $\ell(\nu)$ is the optimal labor demand function in a segment with marginal cost ν. The residual measure of workers is employed in the outside sector $L_o = L-(V-M)-L_P-L_E$.

The economy is endowed with K units of capital. A measure of $K_P = \bar{\kappa}V$ units is employed in the differentiated sector, where $\bar{\kappa} = H(c_D)^{-1}\int_0^{\delta c_D} \ell(\nu)dH(\nu)$ and $\kappa(\nu)$ is the optimal capital demand function in a segment with marginal cost ν. The residual units of capital are employed in the outside sector $K_o = K - K_P$.

Let $\bar{r} = H(c_D)^{-1}\int_0^{c_D} \tilde{r}(\nu)dH(\nu)$ and $\bar{\pi} = H(c_D)^{-1}\int_0^{c_D} \tilde{\pi}(\nu)dH(\nu)$ $\tilde{r}(\nu)$ be the revenue and profit functions in a segment with marginal cost ν. Thus, aggregate revenue and aggregate profit in the differentiated sector are equal to $\bar{r}V$ and $\bar{\pi}V$. Revenue of the outside sector is implied by the production function given the employment of labor and capital $Q_o = L_o + \theta K_o$.

The ownership of firms and factor of production is evenly distributed among consumers. Expenditure in consumption of differentiated good, consumption of outside good and financing firm entry is equal to the sum of labor income, capital income and profit:

$$\bar{r}V + Q_o + f_E M_E = L + \theta K + \bar{\pi}V \ .$$

All consumers earn a per capita income $I^c = (L + \theta K + \bar{\pi}V)/L$. Out of which, $I_d^c = \bar{r}V/L$ is the expenditure in differentiated goods, $I_o^c = Q_o/L$ is the expenditure in the outside good, and the residual $\bar{\pi}V/L$ is allocated to financing the entry of new firms. Individual welfare corresponds to the indirect utility function associated with the consumer's problem.

Welfare

Substituting $q_o^c = Q_o/L = I_o$, for the individual demand $q^c(\nu) = \frac{1}{2\gamma}(p^{max} - \nu)$, for $p^{max} = c_D$ and $Q_c = \int_0^V q^c(\nu) = (\alpha - \bar{p})\frac{V}{\gamma + \beta V}$ in (1) yields individual welfare:

$$W \;=\; I_o + \alpha\frac{(\alpha-\bar{p})V}{\gamma+\beta V} - \frac{1}{2}\frac{V}{\gamma}[(c_D-\bar{p})^2 + \sigma_p^2] - \frac{\beta}{2}(\alpha-\bar{p})^2\left(\frac{V}{\gamma+\beta V}\right)^2$$

where $\int_0^V [c_D - p(v)]\, dv = V(c_D - \bar{p})$ and $\int_0^V [c_D - p(v)]^2\, dv = V(c_D^2 - 2c_D\bar{p} + \bar{\bar{p}})$ and $\sigma_p^2 = \bar{\bar{p}} - \bar{p}^2$ with $\bar{\bar{p}} = H(c_D)^{-1}\int_0^{c_D} p(\nu)^2 dH(\nu)$. Substituting for $\nu \equiv z_i c = 2p(\nu) - c_D$ in the expression of the revenue and dividing by the mass of agents L allows the average individual expenditure in a given variety to be determined. Thus, the individual expenditure in differentiated goods is $I_d^c = \frac{V}{\gamma}[(c_D - \bar{p})\bar{p} - \sigma_p^2]$. Expenditure in the outside good is $I_o = I^c - I_d^c$. Substituting in the expression of welfare yields:

$$W = I^c + \left(\frac{1}{2}\frac{V(\alpha - \bar{p})^2}{\gamma + \beta V}\right) + \frac{\sigma_p^2}{2}\frac{V}{\gamma}$$

where $c_D - \bar{p} \equiv \frac{\gamma}{\gamma + \beta V}(\alpha - \bar{p})$, given the expression for $p^{max} = \frac{1}{\gamma+\beta V}\left(\gamma\alpha + \beta V \bar{p}\right)$.

Firm performances in open economy

Let $\mathbf{1}_{[z_i c]}$ be an indicator function that is $\mathbf{1}_{[z_i c]} = 1$ if there are exports in segment i of a firm c and zero otherwise. Firm performances at the segment level in open economy are:

$$\begin{aligned}
q(z_i c) &= \frac{L}{2\gamma}\left[(c_D - z_i c) + \mathbf{1}_{[z_i c]}(c_D - z_i \tau c)\right] \\
r(z_i c) &= \frac{L}{4\gamma}\left[\left(c_D^2 - (z_i c)^2\right) + \mathbf{1}_{[z_i c]}\left(c_D^2 - (z_i \tau c)^2\right)\right] \\
l(z_i c) &= \frac{z_i c}{\varphi_l}\frac{L}{2\gamma}\left[(c_D - z_i c) + \mathbf{1}_{[z_i c]}(c_D - z_i \tau c)\right] \\
k(z_i c) &= \frac{z_i c}{\varphi_k}\frac{L}{2\gamma}\left[(c_D - z_i c) + \mathbf{1}_{[z_i c]}(c_D - z_i \tau c)\right] \\
\pi(z_i c) &= \frac{L}{4\gamma}\left[(c_D - z_i c)^2 + \mathbf{1}_{[z_i c]}(c_D - z_i \tau c)^2\right] .
\end{aligned}$$

9.1 Drivers of market competition in open economy

The model allows for a tractable analysis of several drivers of market competition in open economy, which are outlined in the following result:

Proposition 2. *The toughness of market competition, measured by a lower cutoff cost* c_D*, is*
(3.1) weaker the greater the trade barriers $\frac{\partial c_D}{\partial \tau} > 0$
(3.2) stronger the greater the market size $\frac{\partial c_D}{\partial L} < 0$
(3.3) weaker the greater the taste for differentiation $\frac{\partial c_D}{\partial \gamma} > 0$
(3.4) weaker the greater the cost of a new entrepreneurial activity $\frac{\partial c_D}{\partial f_E} > 0$
(3.5) stronger the more concentrated the distribution of technology $\frac{\partial c_D}{\partial \rho} < 0$

The proof of Proposition 2 is organized in four steps. First, we determine the expected value of the core segment from sales in the domestic market. Second, we determine the expected value of non-core segments from sales in the domestic market. Third, we determine the expected value of a firm in open economy, thus, accounting for sales in both the domestic and the foreign market; this allows us to solve for the free entry condition (9) in closed form.[26] Finally, we exploit the comparative statics on expected value before entry to prove the comparative statics on the cutoff cost.

We prove a more general version of Proposition 2, which extends the main results in two directions. First, as we did for Proposition 1, we conduct the proof allowing for an arbitrary share of profit $\eta \in [0,1]$. We show that this is redundant since the expected gross profit from non-core divisions does not depend on the profit share, hence MSFs would optimally set $\eta = 0$. Second, we derive the results in open economy allowing for a fixed cost of exporting at the segment level $f_X \geq 0$. This extends the results presented in the main body of the paper.

Core segment. In closed economy, the expected value from sales of the core segment when the market cutoff is c_D unconditional on making a successful entry is:

$$\tilde{\pi} = \int_0^{c_D} \frac{L}{4\gamma} (c_D - c)^2 \, dG(c)$$

Listing the parameters and taking a first order derivative yields:

$$\frac{\partial \tilde{\pi}}{\partial c_D} > 0 \qquad \frac{\partial \tilde{\pi}}{\partial L} > 0 \qquad \frac{\partial \tilde{\pi}}{\partial \gamma} < 0 \ .$$

Non-core segment. The expected value to the firm from non-core divisions is computed on the probability density of the event $m = 1, 2, ...$ *non-core divisions are financed when the maximum customization cost for a profitable division at the firm is* $\bar{z}(c, c_D)$. This is given by the complement of the probability that a manager with reported cost z is still financed at the firm (19), which is what we called $\left(F(z|m)/F(\bar{z}(c, c_D)|m)\right)^m$ in the proof of the Lemma. The expected value from $m = 1, 2, ...$ non-core divisions to a firm with core competence cost c that offer a share of profit $\eta \in [0,1]$ when the market cutoff is c_D is defined as

$$m \left(\int_1^{\bar{z}(c,c_D)} [(1-\eta)\pi(zc) - f_M] \left(\frac{F(z|m)}{F(\bar{z}(c,c_D)|m)} \right)^m \frac{dF(z|m)}{F(z|m)} \right) \ ,$$

[26] In the solution of the free entry condition we generalize existing frameworks (such as Melitz and Ottaviano, 2008 or Mayer et al., 2014) to account for the presence of a fixed cost, which in our model corresponds to the managerial compensation.

which, substituting for $(F(z|m)/F(\bar{z}(c,c_D)|m))^m = F(z)/[mF(\bar{z}(c,c_D))]$ as implied by (7), yields

$$\int_1^{\bar{z}(c,c_D)} \frac{[(1-\eta)\pi(zc) - f_M]dF(z)}{F(\bar{z}(c,c_D))} ,$$

which is the expected value from $m = 1, 2, ...$ non-core divisions of a MSF with maximum customization cost for a profitable division $\bar{z}(c,c_D)$.

Corollary. *By offering a profit share $\eta > 0$ the firm does not improve on the expected profit from non-core divisions.*

Proof. Consider the tournament among applicants to a firm with core competence cost c hiring $m = 1, 2, ...$ divisional managers when the market cutoff is c_D. The firm meets with arbitrarily many managers reporting their customization costs independently. This implies that the reported customization cost by a single manager does not affect the distribution of reported customization costs faced by the firm and other applicants.

Both the MSF and candidate managers know that after the tournament, the reported customization cost in each non-core division will be a random draw from the conditional distribution $F(z)/[mF(\bar{z}(c,c_D))]$, which does not depend on the share of profit η. Therefore, a MSF cannot improve in the expected value from non-core divisions by offering $\eta > 0$. Instead, any positive profit share would decrease the expected value to the firm from non-core divisions. Thus, the optimal choice by a MSF is to offer $\eta = 0$, and this is understood by both firms and managers.■

Based on the previous result, the expected value to the firm from financing a fringe of non-core divisions (23) is given by:

$$\int_1^{\bar{z}(c,c_D)} \frac{[\pi(zc) - f_M]dF(z)}{F(\bar{z}(c,c_D))} .$$

The probability density $\pi(zc)dF(z)/F(\bar{z}(c,c_D))$ has a mass point at the lower extreme of the support $z = 1$. Therefore, the expected value to the firm from a fringe of non-core divisions is given by:

$$\frac{[\pi(c) - f_M]F(1)}{F(\bar{z}(c,c_D))} + \int_{1_+}^{\bar{z}(c,c_D)} \frac{[\pi(zc) - f_M]dF(z)}{F(\bar{z}(c,c_D))} .$$

A firm with $c \leq c_M(c_D)$ meets randomly with managers, and with a probability $F(\bar{z}(c,c_D))$ matches managers with profitable projects. Thus, the *ex-ante* expected value from non-core divisions unconditional on making a successful turn into a MSF is given by:

$$\pi^{nc}(c) = \pi(c)F(1) + \int_{1_+}^{\bar{z}(c,c_D)} \pi(zc)dF(z) - F(\bar{z}(c,c_D))f_M \ .$$

Corollary. *The comparative statics on the ex-ante expected value of non-core divisions* $\tilde{\pi}^{nc}(c)$ *are:*

$$\frac{\partial \pi^{nc}(c)}{\partial c} < 0 \qquad \frac{\partial \pi^{nc}(c)}{\partial c_D} > 0 \qquad \frac{\partial \pi^{nc}(c)}{\partial L} > 0 \qquad \frac{\partial \pi^{nc}(c)}{\partial \gamma} < 0 \ .$$

Proof. To the purpose of facilitating the analysis of comparative statics, define the vector of parameters $\chi = \{c, c_D, L, \gamma\}$, and change the notation as follows $\pi(z;\chi) \equiv \pi(zc)$ and $\bar{z}(\chi) \equiv \bar{z}(c, c_D)$. The ex-ante expected value from non-core divisions in a firm with core competence cost c is given by:

$$\pi^{nc}(c) = \pi(1;\chi)F(1) + \int_{1_+}^{\bar{z}(\chi)} \pi(z;\chi)dF(z) - F(\bar{z}(\chi))f_M \ .$$

Moreover, notice that $\pi(\bar{z}(\chi);\chi) = f_M$ by definition of $\bar{z}(\chi)$. The comparative statics with respect to any $\chi_i = \{c, c_D, L, \gamma\}$ are given by:

$$\frac{\partial \pi^{nc}(c)}{\partial \chi_i} = \frac{\partial \pi(1;\chi)}{\partial \chi_i}F(1) + \int_{1_+}^{\bar{z}(\chi)} \frac{\partial \pi(z;\chi)}{\partial \chi_i}dF(z) \ .$$

Thus, the comparative statics are of the same sign as the corresponding comparative statics of the profit function for the core segment.■

Open economy. Sales to the foreign market are subject to an ad-valorem cost $\tau \geq 1$ and a fixed cost $f_X \geq 0$. The expected value of the core segment from domestic and foreign sales when the market cutoff is c_D unconditional on making a successful entry are:

$$\begin{aligned}
\tilde{\pi}_D &= \int_0^{c_D} \frac{L}{4\gamma}\left(c_D - c\right)^2 dG(c;\boldsymbol{\rho}) \\
\tilde{\pi}_X &= \int_0^{\frac{c_D - \sqrt{\frac{4\gamma f_X}{L}}}{\tau}} \left[\frac{L}{4\gamma}\left(c_D - \tau c\right)^2 - f_X\right] dG(c;\boldsymbol{\rho})
\end{aligned}$$

where $\boldsymbol{\rho}$ is the vector of parameters of the exogenous distribution of core competence costs $G(c)$. Taking the first order derivative yields:

$$\frac{\partial \tilde{\pi}_D}{\partial c_D} > 0 \qquad \frac{\partial \tilde{\pi}_D}{\partial L} > 0 \qquad \frac{\partial \tilde{\pi}_D}{\partial \gamma} < 0 \ , \tag{27}$$
$$\frac{\partial \tilde{\pi}_X}{\partial c_D} > 0 \qquad \frac{\partial \tilde{\pi}_X}{\partial \tau} < 0 \qquad \frac{\partial \tilde{\pi}_X}{\partial f_X} < 0 \qquad \frac{\partial \tilde{\pi}_X}{\partial L} > 0 \qquad \frac{\partial \tilde{\pi}_X}{\partial \gamma} < 0 \ .$$

A segment with marginal cost zc earns a profit in the foreign market equal to $\pi(z\tau c) = \frac{L}{4\gamma}\left(c_D - z\tau c\right)^2$ and $\bar{z}_X(\tau c, c_D)$ is the maximum customization cost such that $\pi(\bar{z}_X(\tau c, c_D)\tau c) = f_X$. Domestic and export cutoff levels of customization costs are:

$$\begin{aligned} \bar{z}_D(c, c_D) &= \frac{c_D - \sqrt{\frac{4\gamma f_M}{L}}}{c} \ , \\ \bar{z}_X(\tau c, c_D) &= \frac{c_D - \sqrt{\frac{4\gamma f_X}{L}}}{\tau c} \ . \end{aligned}$$

To facilitate the analysis of comparative statics we change notation as before $\pi(z\tau;\chi) \equiv \pi(z\tau c)$ and $\bar{z}_X(\tau;\chi) \equiv \bar{z}_X(\tau c, c_D)$, where $\chi = \{c, c_D, L, \gamma\}$. The ex-ante expected value of non-core divisions in a firm with core competence cost c is given by:

$$\begin{aligned} \pi_D^{nc}(c) &= \pi(1;\chi)F(1) + \int_{1_+}^{\bar{z}_D(\chi)} \pi(z;\chi)dF(z) - F(\bar{z}_D(\chi))f_M \\ \pi_X^{nc}(c) &= \pi(\tau;\chi)F(1) + \int_{1_+}^{\bar{z}_X(\chi)} \pi(z\tau;\chi)dF(z) - F(\bar{z}_X(\chi))f_X \ . \end{aligned}$$

The signs of comparative statics on $\pi_D^{nc}(c)$ and $\pi_X^{nc}(c)$ are the same as $\pi_D(c)$ and $\pi_X(c)$. Taking the expectation over the core competence cost yields the unconditional expected value of non-core divisions

$$\begin{aligned} \tilde{\pi}_D^{nc} &= \int_0^{c_D} \pi_D^{nc}(c)dG(c;\boldsymbol{\rho}) \\ \tilde{\pi}_X^{nc} &= \int_0^{\frac{c_D - \sqrt{\frac{4\gamma f_X}{L}}}{\tau}} \pi_X^{nc}(c)dG(c;\boldsymbol{\rho}) \end{aligned}$$

which are characterized by the same comparative statics as (27):

$$\frac{\partial \tilde{\pi}_D^{nc}}{\partial c_D} > 0 \qquad \frac{\partial \tilde{\pi}_D^{nc}}{\partial L} > 0 \qquad \frac{\partial \tilde{\pi}_D^{nc}}{\partial \gamma} < 0 \ , \tag{28}$$
$$\frac{\partial \tilde{\pi}_X^{nc}}{\partial c_D} > 0 \qquad \frac{\partial \tilde{\pi}_X^{nc}}{\partial \tau} < 0 \qquad \frac{\partial \tilde{\pi}_X^{nc}}{\partial f_X} < 0 \qquad \frac{\partial \tilde{\pi}_X^{nc}}{\partial L} > 0 \qquad \frac{\partial \tilde{\pi}_X^{nc}}{\partial \gamma} < 0 \ .$$

We are now in the position to determine the comparative statics on the expected value of a firm before entry, which lead to the proof of Proposition 2.

The expected value of a firm unconditional on making a successful entry is the sum of the two components, from domestic sales and sales in the foreign market:

$$\Pi_D\left(c_D; L, \gamma, \boldsymbol{\rho}\right) = \tilde{\pi}_D + \tilde{\pi}_D^{nc} \qquad\qquad \Pi_X\left(c_D; \tau, f_X, L, \gamma, \boldsymbol{\rho}\right) = \tilde{\pi}_X + \tilde{\pi}_X^{nc} \ ,$$

where we made the parameters explicit. The total expected value of a firm unconditional on making a successful entry is

$$\Pi\left(c_D; \tau, f_X, L, \gamma, \boldsymbol{\rho}\right) = \Pi_D\left(c_D; L, \gamma, \boldsymbol{\rho}\right) + \Pi_X\left(c_D; \tau, f_X, L, \gamma, \boldsymbol{\rho}\right) \ ,$$

whose comparative statics are implied by (27) and (28):

$$\frac{\partial \Pi}{\partial c_D} > 0 \qquad \frac{\partial \Pi}{\partial \tau} < 0 \qquad \frac{\partial \Pi}{\partial f_X} < 0 \qquad \frac{\partial \Pi}{\partial L} > 0 \qquad \frac{\partial \Pi}{\partial \gamma} < 0 \ . \tag{29}$$

Finally, changes in the parameters of the distribution $G(c)$ do not effect the extremes of integration or the integrand functions involved in the determination of the expected value of the firm. Therefore, the comparative statics on the parameters $\boldsymbol{\rho}$ have the same sign as the corresponding comparative statics on the density:

$$sign\left\{\frac{\partial \Pi}{\partial \rho_i}\right\} = sign\left\{\frac{\partial^2 G(c;\boldsymbol{\rho})}{\partial^2 \rho_i}\right\} \ , \tag{30}$$

for every $\rho_i \in \boldsymbol{\rho}$. In the main body of the paper we refer to one parameter ρ, instead of a vector. However, the increase of any parameter $\rho_i \in \boldsymbol{\rho}$ which determines *ceteris paribus* a higher probability density implies a greater concentration in the distribution of core competence cost. Thus, the interpretation of the comparative statics with respect to concentration comes without loss of generality.

From (9) it is immediate to conclude that for $c_D = 0$ the value Π is null as well. Therefore, for every $f_E > 0$, the there exists one and only one $c_D^\star$ such that $\Pi(c_D) < f_E$ for $c_D < c_D^\star$ and $\Pi(c_D) > f_E$ for $c_D > c_D^\star$. An analysis of the intersection $\Pi(c_D^\star) = f_E$ given (29) determines the comparative statics on the market cutoff:

$$\begin{aligned} &\frac{\partial c_D^\star}{\partial \tau} > 0 \quad \frac{\partial c_D^\star}{\partial f_X} > 0 \quad \frac{\partial c_D^\star}{\partial L} < 0 \quad \frac{\partial c_D^\star}{\partial \gamma} > 0 \\ &\frac{\partial c_D^\star}{\partial f_E} > 0 \quad sign\left\{\frac{\partial c_D^\star}{\partial \rho_i}\right\} = -sign\left\{\frac{\partial^2 G(c;\boldsymbol{\rho})}{\partial^2 \rho_i}\right\} \ . \end{aligned} \tag{31}$$

Endogenous sources of heterogeneity

In this section, we show how the introduction of an internal capital market allows us to endogenise the cost structure of multi-segment firms, which remains exogenous in the multi-product-firms literature.

Managers strategically over-report the cost of their divisions depending on the true customization cost of the division x_i; the core marginal cost of the firm c; the number of non-core divisions m; and competition in the output market, through c_D (Proposition 1). Consequently, the cost structure of a firm becomes a distinct feature of the firm's organisation: the firm's organisation acts as a glue. The cost of a segment depends not only on its customization costs but also on the competition for funds in the internal capital market and the number of segments a firm operates.

To illustrate this point, we compare the relative cost across divisions in Mayer et al. (2014) with our setup. In their model, the relative marginal cost between two products of the same firm, say i and j, is exogenously determined by the ratio of their customization costs. In our model, the relative marginal cost between two products becomes endogenous to the four determinants of misreporting

$$\frac{z_i c}{z_j c} = \frac{\mu^\star(x_i, c, m, c_D)}{\mu^\star(x_j, c, m, c_D)} \frac{x_i}{x_j} \neq \frac{x_i}{x_j} ,$$

where the strategic over-reporting by the two managers generates a distortion to the allocation of capital. Because the allocation of capital is increasing in the reported marginal cost, both non-core divisions are characterised by an excess of capital $k(z_i, c) > k(x_i, c)$ and $k(z_j, c) > k(x_j, c)$.

This property extends to a comparison across firms. The marginal costs of two products with the same customization cost but produced in different firms, say with core marginal costs c' and c'', and/or with m' and m'' non-core divisions:

$$\frac{z_i c'}{z_i c''} = \frac{\mu^\star(x_i, c', m, c_D)}{\mu^\star(x_i, c'', m, c_D)} \frac{c'}{c''} \neq \frac{c'}{c''} \quad \text{or} \quad \frac{z_i c'}{z_i c''} = \frac{\mu^\star(x_i, c', m', c_D)}{\mu^\star(x_i, c'', m'', c_D)} \frac{c'}{c''} \neq \frac{c'}{c''},$$

are no longer exogenous as in Mayer et al. (2014). Our model predicts that more capital is allocated to the division of the better firm (lower c) and, given the same core competence cost, to the division of the firm with a wider number of non-core segments (greater m).

Moreover, the result that changes in competition (such as a lower output market cutoff c_D) propagates through the cost structure of multi-segment firms and is a distinctive feature of our approach, due to the strategic behavior of managers as captured by the endogenous factor of misreporting $\mu^\star(x_i, c, m, c_D)$. Through this channel the policies that affect the output market have an heterogeneous impact on products and firms.